

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 10039

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Unique Paper Code : 2353200017

Name of the Paper : Applied Algebra

Name of the Course : **NEP UGCF-2022 BACHELOR OF ARTS/
BACHELOR OF SCIENCE (PROGRAMME)**

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are compulsory.
3. Attempt any two parts from each question.

1. (a) An economy consists of three sectors: Coal, Electric (power), and Steel. The distribution of output of each sector among all the sectors is given below:

Purchased By/Produced By	Coal	Electric	Steel
Coal	0.0	0.4	0.6
Electric	0.6	0.1	0.2
Steel	0.4	0.5	0.2

Let P_C, P_E, P_S denote the prices of the total annual outputs of the Coal, Electric and Steel sectors respectively.

- (i) Form a system of linear equations representing the condition that income of each sector equals its expenditure.
- (ii) Solve the system using Row Reduction Method.
- (iii) Find the general equilibrium price vector.
- (iv) Interpret the economic meaning of your solution.

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- (b) Consider an economy consisting of three sectors: Manufacturing, Agriculture, and Services. The consumption matrix of the economy is given by

$$C = \begin{bmatrix} 0.50 & 0.40 & 0.20 \\ 0.20 & 0.30 & 0.10 \\ 0.10 & 0.10 & 0.30 \end{bmatrix}$$

Suppose the final demand vector is $d = \begin{bmatrix} 50 \\ 30 \\ 20 \end{bmatrix}$

- (i) Write the Leontief input-output model for this economy.
 - (ii) Form the system of linear equations corresponding to the model.
 - (iii) Solve for the production vector x using matrix method.
 - (iv) Interpret the economic meaning of the production vector obtained.
- (c) Define homogeneous coordinates and explain their importance in computer graphics. Show how a translation

$$(x, y) \rightarrow (x + h, y + k)$$

can be represented using matrix multiplication in homogeneous coordinates.

2. (a) Verify that $y_k = k^2$ is a solution of the difference equation

$$y_{k+2} - 4y_{k+1} + 3y_k = -4k, \forall k$$

Find the general solution of the equation.

- (b) Consider a system whose state is described by a Markov Chain:

$$x_{k+1} = Px_k, \quad k = 0, 1, 2, \dots$$

where the transition matrix is

$$P = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.8 & 0.3 \\ 0.2 & 0 & 0.4 \end{bmatrix}$$

and the initial state vector is

$$x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Compute the state vectors x_1, x_2, x_3 and continue the process to obtain x_k for sufficiently large k (upto $k = 10$).

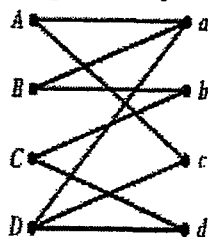
- (c) Find a least-square solution of the system $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

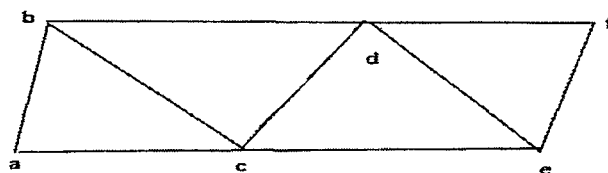
3. (a) Show that the Latin square illustrated in the table cannot be obtained from a group table.

A	B	C	D	E
B	A	E	C	D
C	D	A	E	B
D	E	B	A	C
E	C	D	B	A

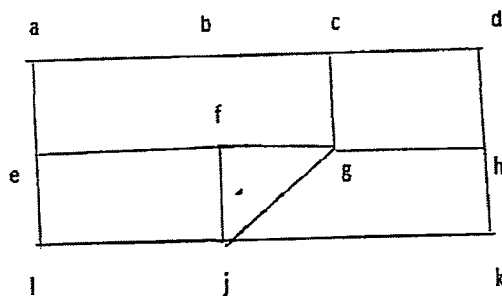
- (b) Find a matching or explain why none exists for the following graph:



- (c) Find a maximal independent set in the following graph:



4. (a) Find a minimal edge cover for the following graph. Give a detailed logical analysis.



- (b) Define a Latin square. Give examples. Construct a Latin square of order 7 on $\{a, b, c, d, e, f, g\}$.
- (c) Define an independent set and an edge cover in a graph. Give examples. State a relationship between an independent set and an edge cover in a graph.
5. (a) The US postal money order check digit scheme detects errors in identification Numbers. Discuss whether this scheme detects all single-digit errors. If not, identify the cases where it fails and explain the mathematical reason behind the failure.
- (b) Using the UPC check digit scheme, assign a check digit to the number 3 – 88207 – 29431. In which cases does the UPC check digit scheme fail to catch transposition-of-adjacent-digits errors? Illustrate with examples.
- (c) Define the ten-digit ISBN check digit scheme. Explain mathematically why this scheme catches all single-digit errors.
6. (a) Define a rigid motion. Show that translation is a rigid motion. Is every rigid motion a translation? Justify your answer with an example.
- (b) Define a regular polygon. Examine the ten symmetries of a pentagon in detail.
- (c) Define the Verhoeff check digit scheme. Show that the identification number 57014 is an invalid number using the Verhoeff scheme.