[This question paper contains 4 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper : **7388**

Unique Paper Code : 2353200020

Name of the Paper : Optimization Techniques

Name of the Course : **Bachelor of Arts/Bachelor of Science (Programme)**

Semester : VIII

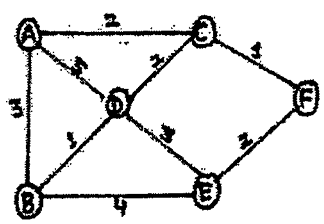
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator (not scientific) is allowed.

1(a)



To develop forward recursive equation and use it find the shortest path from A to E.

(7.5)

1(b) Solve the following Cost Minimizing Transportation problem.

	44	13	15
24	2	1	3
14	4	6	5
34	8	9	7

(7.5)

1(c) Find the optimal solution of the fractional knapsack problem with M=15, n=7

Object	1	2	3	4	5	6	7
Wright	1	3	5	4	1	3	2
Profit	5	10	15	7	8	9	4

(7.5)

P.T.O.

2(a) Consider the following transshipment problem with two sources and destinations, the costs for shipment in rupees are given below. Determine an optimal shipment schedule:

		Source		Destination		Supply
		S_1	S_2	D_1	D_2	
Source	S_1	0	1	6	2	4
	S_2	1	0	8	2	5
Destination	D_1	0	7	0	3	–
	D_2	1	0	1	0	–
	Demand	–	–	3	6	

(7.5)

2(b) In a cargo-loading problem, there are three items of different per unit weight and revenue as given below

Item i	w_i(ton)	r_i($)
1	4	70
2	1	20
3	2	40

Maximum cargo loaded is restricted to 8 tons. The goal is to determine the number of units of each item that will maximize the total return. (7.5)

2(c) State the 'Principle of optimality' in dynamic programming and give a mathematical formulation of a dynamic programming problem. (7.5)

3 (a) Solve the integer linear programming problem graphically

$$Max\ z = x_1 - x_2$$

$$subject\ to\ \ x_1 + 2x_2 \leq 4\ ,$$

$$6x_1 + 2x_2 \leq 9\ ;$$

$$x_1, x_2 \geq 0,\ x_1\ and\ x_2\ integers$$ (7.5)

3(b) Solve the integer linear programming problem using Gomory's cutting plane methods

$$Max\ z = x_1 + x_2$$

$$subject\ to\ \ x_1 + 2x_2 \leq 4\ ,$$

$$x_2 \leq 2\ ;$$

$$x_1, x_2 \geq 0,\ x_1\ and\ x_2\ integers$$ (7.5)

3 (c) Solve the mixed integer linear programming problem

$$Max\ z = 2x_1 + x_2$$
$$subject\ to\ \ x_1 + x_2 \leq 5$$
$$6x_1 + 2x_2 \leq 21$$
$$x_1, x_2 \geq 0\ \ x_1\ is\ an\ integer \qquad (7.5)$$

4(a) Solve the mixed integer linear programming problem

$$Min\ z = x_1 - 3x_2$$
$$subject\ to\ \ x_1 + x_2 \leq 5$$
$$-2x_1 + 4x_2 \leq 11$$
$$x_1, x_2 \geq 0\ \ and\ x_2\ is\ an\ \ integer \qquad (7.5)$$

4(b) Use branch and bound method to solve the integer linear programming problem

$$Max\ z = 3x_1 + 4x_2$$
$$subject\ to\ \ 7x_1 + 16x_2 \leq 52,$$
$$3x_1 - 2x_2 \leq 9;$$
$$x_1, x_2 \geq 0, x_1\ and\ x_2\ \ integers \qquad (7.5)$$

4(c) Use branch and bound method to solve the integer linear programming problem

$$Max\ z = x_1 + 2x_2$$
$$subject\ to\ \ x_1 + 2x_2 \leq 12,$$
$$4x_1 + 3x_2 \leq 14;$$
$$x_1, x_2 \geq 0,\ \ x_1\ and\ x_2\ \ integers. \qquad (7.5)$$

5(a) Define concavity of a function. Let S be a non-empty convex set in $\mathbb{R}$. Examine the concavity/convexity of the function

$$f(x) = 6x - x^2 - 3\ \text{ for } x \in \mathbb{R}. \qquad (7.5)$$

5(b) Let S be a non-empty convex subset of $\mathbb{R}^n$ and let $f : S \to \mathbb{R}$ be a convex function and $\alpha \geq 0$. Then, prove that $h(x) = \alpha f(x)$ is a convex function. (7.5)

5(c) Let $S \subseteq \mathbb{R}^3$ be a convex set and $f : S \to \mathbb{R}$ be given by

$f(x_1, x_2, x_3) = -x_1^2 + 2x_1x_2 + 3x_1x_3 + 6x_2x_3$, $(x_1, x_2, x_3) \in \mathbb{R}^3$.

Examine the convexity/concavity of f. (7.5)

6(a) Is the following problem a convex programming problem? Give reasons for your answer.

$$\begin{aligned} &Max \ \ x_2 \\ &\text{subject to } x_1^2 + x_2^2 \leq 1 \\ &\qquad x_1^2 \geq x_2 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Also, show it geometrically. (7.5)

6(b) Solve the following linear fractional programming problem by the Charnes and Cooper Method

$$\begin{aligned} &Max \ \ z = \frac{2x_1 + x_2}{-x_1 - 2x_2} \\ &\text{subject to } x_1 + x_2 \leq 6 \\ &\qquad 2x_1 + x_2 \geq 4 \\ &\qquad x_1, x_2 \geq 0. \end{aligned}$$

(7.5)

6(c) Solve the following linear fractional programming problem by the simplex Algorithm:

$$\begin{aligned} &Max \ \ z = \frac{2x_1 + 3x_2}{x_1 + x_2 + 7} \\ &\text{subject to } 3x_1 + 5x_2 \leq 15 \\ &\qquad 4x_1 + 3x_2 \leq 12 \\ &\qquad x_1, x_2 \geq 0. \end{aligned}$$

(7.5)