[This question paper contains 2 printed pages.]

Your Roll No................

Sr. No. of Question Paper	**:**	**7386**	**L**
Unique Paper Code	:	2353200018	
Name of the Paper	:	Elements of Partial Differential Equations	
Name of the Course	:	**DSE-Bachelor of Arts / Bachelor of Science (Programme)**	
Semester	:	VIII	

Duration : 3 Hours Maximum Marks : 90

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer any **two** parts from each question.
3. **All** questions carry equal marks.

Q.1 (a) Find the integral surface of the linear partial differential equation
$$x(y^2+z)\frac{\partial z}{\partial x} - y(x^2+z)\frac{\partial z}{\partial y} = (x^2-y^2)z$$

(b) Find the solution to the equation $z = \frac{1}{2}(p^2+q^2)+(xp = yq, y^2 = z(xp+yq) = 2xy$.

(c) Find the general solution to the differential equation
$$x(z+2a)\frac{\partial z}{\partial x} + (xz+2yz+2ay)\frac{\partial z}{\partial y} = z(z+a).$$

Q.2 (a) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$. Orthogonally and which passes through the circle $x^2+y^2=1, z=1$.

(b) Find the solution to the equation $(y+zx)u_x - (x+yz)u_y = x^2 - y^2$

(c) Find the solution to the equation $z = \frac{1}{2}(p^2+q^2)+(p-x)(q-y)$. which passes through the x-axis.

Q.3 (a) Find the solution to the characteristic equations $pq = z$ and determine the integral surface which passes through the parabola $x=0, y^2 = z$

(b) Show that the equations $xp = yq$, $z(xp+yq) = 2xy$ are compatible and solve them.

(c) If $z = f(x^2 - y) + g(x^2 + y)$ where the functions f,g are arbitrary, prove that

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{x}\frac{\partial z}{\partial x} = 4x^2 \frac{\partial^2 z}{\partial y^2}$$

Q.4 **(a)** Show that if f and g are arbitrary functions of a single variable, then

$u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \quad \text{provided that } \alpha^2 = 1 - \frac{v^2}{c^2}$$

(b) Find the solution of $(D - D' - 1)(D - D' - 2)z = e^{2x-y}$

(c) Solve using Monge's method

$$(q^2 - 1)zr - 2pqzs + (p^2 - 1)zt + z^2(rt - s^2) = p^2 + q^2 - 1$$

Q.5 **(a)** Find the solution of $4D^2 + 12DD' + 9D'^2)z = 0$

(b) Find the solution of $r + 3s + t + (rt - s^2) = 1$

(c) Determine the solution of wave equation with non-homogeneous boundary -value problem

$$u_{tt} - 4u_{xx} = 0, \quad x > 0,\ t > 0,$$
$$u(x,0) = \log(1+x),\ x \geq 0,$$
$$u_t(x,0) = 1,\ x \geq 0,$$
$$u(0,t) = t,\ t \geq 0,$$

Q6 **(a)** Determine the solution of the initial-value problem for a semi-infinite string with fixed end

$$u_{tt} - 4u_{xx} = 0, \quad 0 < x < \infty,\ t > 0,$$
$$u(x,0) = x^4,\ 0 \leq x < \infty,$$
$$u_t(x,0) = 0,\ 0 \leq x < \infty,$$
$$u(0,t) = 0, \quad 0 \leq t < \infty.$$

(b) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$$u_{tt} = c^2 u_{xx} + h(x,t),\ x \in \mathbb{R},\ t > 0$$

with initial condition $u(x,0) = f(x),\ u_t(x,0) = g(x),\ x \in \mathbb{R}.$

(c) Find the solution of wave equation with non-homogeneous boundary conditions:

$$u_{tt} - c^2 u_{xx} = 0, \quad x > 0,\ t > 0,$$
$$u(x,0) = f(x),\ u_t(x,0) = g(x),\ x \geq 0,$$
$$u(0,t) = p(t), \quad t \geq 0.$$