

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 7385

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Unique Paper Code : 2353200017

Name of the Paper : Applied Algebra

Name of the Course : **NEP UGCF-2022 BACHELOR OF ARTS/
BACHELOR OF SCIENCE (PROGRAMME)**

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are compulsory.
3. Attempt any two parts from each question.

1. (a) An economy consists of two sectors: Goods and Services.
Each year:
Goods sector sells 80% of its output to Services and retains the rest
Services sector sells 70% of its output to Goods and retains the rest.
 - (i) Construct the exchange table for this economy.
 - (ii) Form a system of linear equations representing the condition that income equals expenditure for each sector.
 - (iii) Solve the system to find the equilibrium prices of the outputs of Goods and Services.
 - (iv) Interpret your solution economically.

- (b) Solve the Leontief Production equation for an economy with three sectors, given that:

$$\text{Consumption matrix } C = \begin{bmatrix} 0.2 & 0.2 & 0.0 \\ 0.3 & 0.1 & 0.3 \\ 0.1 & 0.0 & 0.2 \end{bmatrix} \text{ and final demand vector } d = \begin{bmatrix} 50 \\ 30 \\ 20 \end{bmatrix}$$

- (c) Use matrix multiplication to find the image of the triangle with data matrix

$$D = \begin{bmatrix} 5 & 2 & 4 \\ 0 & 2 & 3 \end{bmatrix}$$

under the transformation that reflects points through the y-axis. Sketch or give the vertices of both the original triangle and its image.

P.T.O.

2. (a) Solve the difference equation

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0$$

- (i) Find the auxiliary equation.
- (ii) Determine the general solution.
- (iii) State the dimension of the solution space.

- (b) Let $P = \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix}$ be the stochastic matrix.

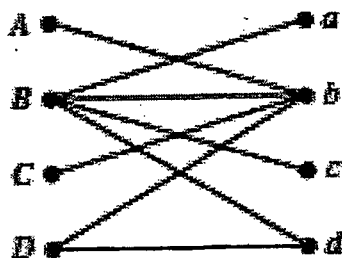
- (i) Find all vectors x such that $Px = x$.
- (ii) Determine a steady-state vector for P and interpret the result.

- (c) Find a least-square solution of the inconsistent system $Ax = b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

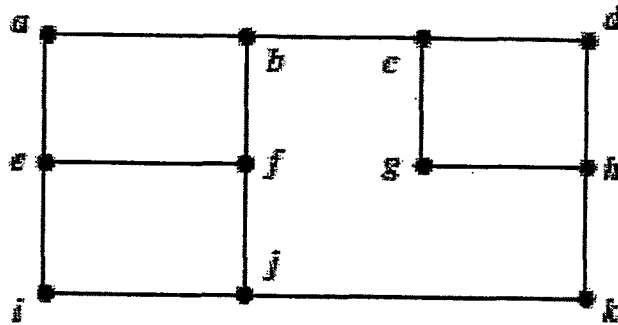
3. (a) A drug company wishes to test four different pain relievers A, B, C and D on four groups of objects each day from Monday to Thursday. Use Latin Square to design an experiment so as to reduce the effects due to difference between the objects and different days.

- (b) Find all possible matching jobs or explain why none exists:

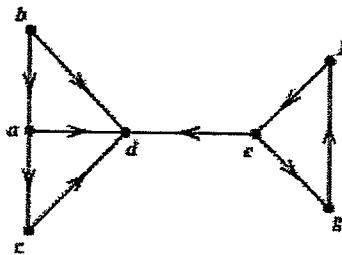


- (c) Three pitchers of sizes 10L, 4L and 7L are given. If initially 10L pitcher is full and other two empty, find a minimal sequence of pouring so as to have exactly 2L in either the 7L or the 4L pitcher.

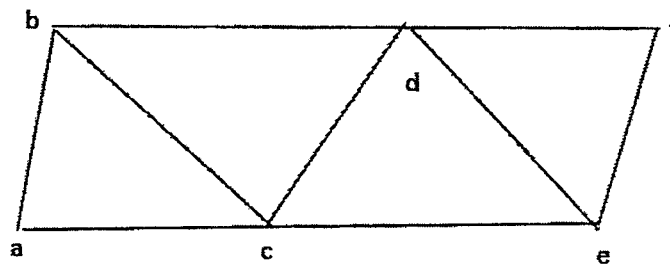
4. (a) The following figure represents a section of city's street map. We want to position police at corners (vertices) so that they can keep every block (edge) under surveillance i.e., every edge should have a policeman at least one of its corners. What is the smallest number of police that can do this job?



- (b) Find a vertex basis for the following graph. Justify your answer.



- (c) Find a minimal edge cover for the graph:



5. (a) (i) Suppose the prime factorization of an integer n contains the prime factors 2, 3 and 11. That is, $n = 2^r 3^s 11^t$ for positive integers $r \geq 1, s \geq 1$ & $t \geq 1$.

Show that (1) 6 divides n

(2) 22 divides n

(3) 7 does not divide n

(4) 35 does not divide n

- (ii) Calculate $(12.18) \pmod{6}$ and $33^7 \pmod{10}$
- (b) Explain how the check digit is computed in the US postal money order system. Verify whether 10962801047 is valid using the US postal money order check digit scheme.
- (c) The number 0 – 037 – 1222494315 is used to identify an Airplane ticket. Use the Airplane Ticket check digit scheme to assign a check digit to this number and write out the entire document number.
6. (a) Define the symmetry and rigid motion of an object. What are the different types of rigid motions. Explain with the help of examples.
- (b) Define a regular polygon. Examine the six symmetries of a triangle in detail.
- (c) Define the Verhoeff check digit scheme. Show that the identification number 3170092 is a valid number using the Verhoeff scheme.