[This question paper contains 2 printed pages.]

Your Roll No................

Sr. No. of Question Paper : 7381 **L**

Unique Paper Code : 2353010023

Name of the Paper : DSE-6(iii) : Industrial Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting three parts from **Q1.-Q4.** and **two** parts from **Q5.-Q6**.
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks. **All** parts of a question carry equal marks. Marks are indicated.

Q1. (a) Define the Laplace transform of a continuous time system $f(t)$. Find the Laplace transform of $f(t) = e^{-at}, t \geq 0$, where a is a constant. (5)

(b) Find the inverse-Laplace transform of $F(s) = \frac{s+5}{(s+1)(s+3)}$. (5)

(c) Define transfer function of a system. Find the transfer function of a system with equation $M\ddot{x} + B\dot{x} + Kx = p$. (5)

(d) Find the state-space model of a system with equation $\ddot{x} + 2\dot{x} + x = 3$. (5)

Q2. (a) Define signal flow graph of a system. Explain signal flow graph of a series system. (5)

(b) Find Z transform of $f(k) = k^2, k \in \mathbb{N}$. (5)

(c) Find the root locus if $G(s)H(s) = \frac{1}{s(s+4)(s^2+4s+20)}$. (5)

(d) Determine whether the characteristic equation $D(s) = s^4 + 6s^3 + 23s^2 + 40s + 50 = 0$ represents a stable system. (5)

Q3. (a) Define signal energy. Find the signal energy of $g(t) = 2e^{-\frac{t}{2}}, t \geq 0$. (5)

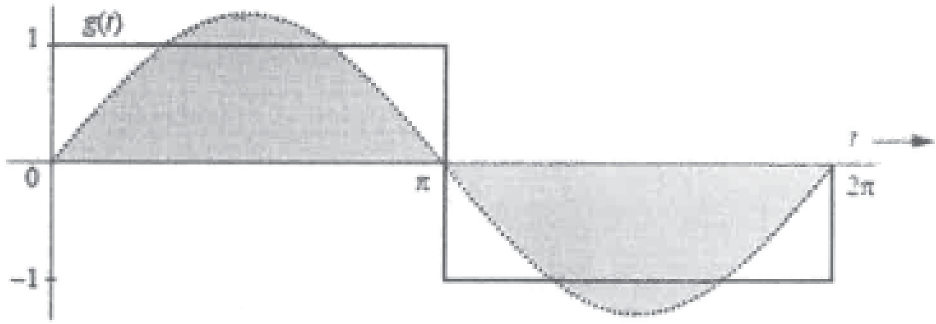
(b) Find the power of the signal $g(t) = 5\cos(6t + 45^0)$. (5)

(c) Explain any one classification of signals with examples. (5)

(d) Consider the signal $g(t) = \begin{cases} t, 0 \leq t \leq 1 \\ 3 - 2t, 1 \leq t \leq 2 \\ t^2 - 5, 2 \leq t \leq 3 \end{cases}$. Sketch time scaled signals $g(2t)$ and $g(\frac{t}{3})$. (5)

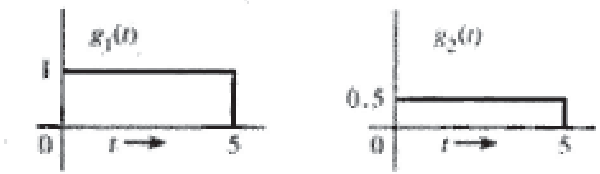
Q4. (a) What is a unit impulse signal. Explain sampling property of unit impulse function. (5)

(b) Express the following signal in term of $\sin(t)$



(5)

(c) Find the correlation coefficient between pulse $g_1(t)$ and $g_2(t)$ given below



(5)

(d) Define orthogonal and orthonormal signals. Give one example of each. (5)

Q5. (a) Obtain the boundary condition representing the absorption of latent heat. (7.5)

(b) Find the general solution of the equation $-\frac{1}{2}\eta\frac{df}{d\eta} = \frac{d^2f}{d\eta^2}$. (7.5)

(c) Show that the substitution $u(x,t) = f(\eta)$, $\eta = xt^{-\frac{1}{2}}$ reduces the partial differential equation $\frac{\partial u}{\partial t} = \alpha\frac{\partial^2 u}{\partial x^2}$ to an ordinary differential equation. (7.5)

Q6. (a) Given the stretching transformation $x = e^a x_*, t = e^b t_*, u = e^c u_*$, express $\frac{\partial^3 u}{\partial x \partial t^2}$ and $\frac{\partial u}{\partial x}\frac{\partial u}{\partial t}$ in the starred variables. (7.5)

(b) Sketch a bifurcation diagram for equilibrium solutions of $\frac{du}{dt} = e^u - \lambda u$. (7.5)

(c) Consider the Fourier series $f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(\frac{n\pi x}{L}) + \sum_{n=1}^{\infty} B_n \sin(\frac{n\pi x}{L})$. Show that $A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$. (7.5)