[This question paper contains 4 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper : 7464

Unique Paper Code : 2373570010

Name of the Paper : DSE : Multivariate Data Analysis

Name of the Course : **B.A. / B.Sc. (P) under NEP-UGCF**

Semester : VIII

Duration : 3 Hours

Maximum Marks : 90

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all, selecting **Three** from each section.
3. Use of non-programmable scientific Calculator allowed.

SECTION A

1. (a) Determine the parameters for the BVN distribution

$$f(x,y) = c\, exp\left\{-[16(x-2)^2 - 12(x-2)(y+3) + 9(y+3)^2]/216\right\}.$$

Also find the value of c and the mean and variance of the conditional distribution of Y|X=x.

(b) For $(X,Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, prove that X and Y are independent iff $\rho = 0$.

(8,7)

2. (a) If $(X,Y)\sim BVN(0,0,1,1,\rho)$ show that moments obey the recurrence relation given by

$$\mu_{rs} = (r+s-1)\rho\mu_{r-1,s-1} + (r-1)(s-1)(1-\rho^2)\mu_{r-2,s-2}\,.$$

Hence show that $\mu_{31} = 3\rho$ and $\mu_{22} = 1 + 2\rho^2$.

(b) Let $(X,Y)\sim BVN(0,0,1,1,\rho)$, then find $P[XY > 0]$. (8,7)

3. (a) If X and Y are standard normal variates with coefficient of correlation ρ between them, then show that $X+Y$ and $X-Y$ are independently distributed.

(b) If $(X,Y)\sim BVN(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$, then obtain the marginal distributions of X and Y.

(8,7)

4. (a) If $\underset{\sim}{X}$ have a *p-variate* normal distribution $N_p(\underset{\sim}{\mu},\Sigma)$ and if $\underset{\sim}{X} = (\underset{\sim}{X_1},\underset{\sim}{X_2})$, where $\underset{\sim}{X_1}$ is a $(q\times 1)$ and $\underset{\sim}{X_2}$ is a $((p-q)\times 1)$ sub-vector, obtain conditional distribution of $\underset{\sim}{X_1}$ given $\underset{\sim}{X_2} = \underset{\sim}{x_2}$.

Hence show that the mean of $\underset{\sim}{X_1}$ given $\underset{\sim}{X_2} = \underset{\sim}{x_2}$ is linear function of given $\underset{\sim}{X_2}$ and covariance matrix of $\underset{\sim}{X_1}$ given $\underset{\sim}{X_2} = \underset{\sim}{x_2}$ does not dependent on $\underset{\sim}{x_2}$.

(b) If $\underset{\sim}{X}$ have a p-variate normal distribution $N_p(\underset{\sim}{\mu},\Sigma)$ and D is $p\times p$ singular matrix then show that $D\underset{\sim}{X} \sim N_p(D\mu, D\Sigma D')$. (8,7)

SECTION B

5. (a) If $\underset{\sim}{X}$, $\underset{\sim}{\mu}$ and Σ are partitioned as $\underset{\sim}{X} = \begin{pmatrix} \underset{\sim}{X}^{(1)}_{q\times 1} \\ \underset{\sim}{X}^{(2)}_{(p-q)\times 1} \end{pmatrix}$, $\underset{\sim}{\mu} = \begin{pmatrix} \underset{\sim}{\mu}^{(1)} \\ \underset{\sim}{\mu}^{(2)} \end{pmatrix}$ and $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$,

then show that $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ are independently distributed if $\Sigma_{12} = 0 = \Sigma_{21}$.

(b) Let $\underset{\sim}{X}$ follows $N_4(\mu, \Sigma)$ with $\mu' = [-3, 1, 4, 2]$ and

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 & 1 \\ -2 & 5 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & -1 & 0 & 3 \end{bmatrix}$$

Find the marginal distribution of each of the random vectors

(i) $V_1 = \frac{1}{4}X_1 - \frac{1}{4}X_2 + \frac{1}{4}X_3 - \frac{1}{4}X_4$

(ii) $V_2 = \frac{1}{4}X_1 + \frac{1}{4}X_2 - \frac{1}{4}X_3 - \frac{1}{4}X_4$ (8,7)

6. (a) Derive the moment generating function of $\underset{\sim}{X}$, follows a p-variate normal distribution $N_p(\underset{\sim}{\mu}, \Sigma)$.

(b) Consider the random vector $X' = X_1, X_2, \ldots, X_5$ having the following covariance matrix.

$$\Sigma = \begin{bmatrix} 1 & -1 & 0.5 & -0.5 & 0 \\ -1 & 3 & 1 & -1 & 0 \\ 0.5 & 1 & 6 & 1 & -1 \\ -0.5 & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$

Find value of $R^2_{2.45}$. (9,6)

7. Describe the steps involved in performing principal component analysis on a given data set. Consider the covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 4 \\ 4 & 100 \end{bmatrix}.$$

Derived the correlation matrix $\boldsymbol{\rho}$ corresponding to above matrix and prove that principal components obtained from $\boldsymbol{\Sigma}$ and $\boldsymbol{\rho}$ are different. Also, calculate the proportion of the total population variance explained by the first principal component for both the matrices.

(15)

8. (i) Explain the concept of Factor Analysis.

(ii) Derive the discriminating procedure in the case of two p-variate normal population with different mean vectors but same dispersion matrix.

(6,9)