[This question paper contains 2 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7461 **L**

Unique Paper Code : 2373010021

Name of the Paper : Advanced Stochastic Processes

Name of the Course : **B.Sc. (H) Statistics**

Semester : VIII (NEP-UGCF)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all. Question no. **1** is compulsory.
3. Attempt **five** questions from the remaining questions.
4. Use of non-programmable scientific calculator is allowed.

Q1. Attempt any *three* parts:

a) State without proof the necessary and sufficient condition for the existence of a unique stationary distribution for a finite Markov chain.

b) Find the characteristic function of a Poisson random variable with parameter λ. Use it to prove that the sum of two independent Poisson random variables is Poisson.

c) Define a Birth-and-Death Process with birth rates λ_n and death rates μ_n. Write the forward Kolmogorov equations.

d) Explain the concepts of average system length (L_s), average queue length (L_q), and average waiting time (W_q), in the context of queueing models.

(3x5=15)

Q2(a) Define aperiodicity and irreducibility for a Markov chain. Prove that for a finite, irreducible, aperiodic Markov chain, the limiting distribution π exists and satisfies $\pi = \pi P$ and $\sum_j \pi_j = 1$.

(b) A store stocks an item with weekly demand $d \in \{0, 1, 2\}$ and probabilities p_0, p_1, p_2. The replenishment policy is, if stock reaches 0 then replenish to capacity S = 2 (order 2 new items) and if stock is positive then do not replenish. Let X_n = stock level at end of n^th week (after demand and replenishment). Find the long-run proportion of weeks at each stock level using Solberg's formula. ($p_0 = 0.3, p_1 = 0.5, p_2 = 0.2$).

(8,7)

Q3(a) How can we calculate P^n, when the eigen values of stochastic matrix are repeated, clearly defining canonical representation, structure and power of the Jordan block.

(b) A child can either go by a car or catch the school bus to go to his school every day. He never goes two days in a row in school bus, but if he goes by bus one day, then the next day he is equally likely to travel by car as he is to go by school bus again. On the first day of the week, he throws a die and goes by bus to his school iff "5 " appears. Find the probability that

(i) he goes by car to his school on the second day of the week.

(ii) he catches a bus in the long run. (8,7)

Q4(a) Define Laplace and Inverse Laplace transform. Find the Laplace transform of $f(t) = t^2e^{-3t}$. Also find the inverse Laplace transform of $F(s) = \frac{2}{(s+3)^2}$.

(b) Consider G.W. branching process $\{X_n, n = 0,1,2, \ldots\}$ with mean m. Show that if $m \leq 1$, the probability of ultimate extinction is 1. If $m > 1$, the probability of ultimate extinction is the positive root less than unity of the equation $s = P(s)$. (7,8)

Q5(a) Define extinction in a branching process. Let

$$p_k = \begin{cases} \mu > 0; & k = 0 \\ 1 - \mu - \lambda > 0; & k = 1 \\ \lambda > 0; & k = 2 \end{cases}$$

Find the probability of ultimate extinction.

(b) Define the generating function of renewal interval T. Find the probability of rth interval at time n. With usual notions, prove that $\sum_n f_n^{(r)} = F^{(r)}(1)$. (8,7)

Q6(a) Obtain the differential-difference equation for the Birth and Death process. Show that for the linear growth process, the mean population size satisfies the differential equation $M'(t) = (\lambda - \mu)M(t)$. Also find $M(t)$ assuming that the population size at $t = 0$ is i.

(b) Write down the differential-difference equations for the Yule-Furry process. Obtain an expression for the size distribution ($p_n(t) = P(N(t) = n)$), assuming that the process starts with one member at time $t = 0$. Further, obtain the probability generating function of the process. (8,7)

Q7(b) Consider $(M/M/1)$: $(FIFO\ /\infty)$ queuing model. Find

(i) The probability that number of customers in the system is $\geq k$.

(ii) Expected number of customers in the queue.

(ii) Average length of nonempty queue.

(b) At what average rate must a server at a counter work in order to ensure that a customer, on average, will spend 12 minutes if the system is (M/M/1) with infinite system capacity and FCFS service discipline. Arrival rate is 15 customers per hour. Find the probability that a customer will have to spend more than 12 minutes in the system. (8,7)

(500)