[This question paper contains 4 printed pages.]

Your Roll No.------------------------

Serial Number of Question Paper	:	**7440**
Unique Paper Code	:	**2223010038**
Name of the Paper	:	**Adv. Quantum Mechanics-II**
Name of the Course	:	**B.Sc. Hons.-(Physics) NEP: UGCF-2022**
Semester	:	**VIII- Semester (DSE Paper)**

Duration : 3 Hours **Maximum Marks: 90**

Instructions

i. Write your Roll Number on the top immediately on the receipt of the question paper.

ii. Attempt five questions in all. Question number 1 is compulsory. Attempt any four questions amongst questions 2-6.

iii. All questions carry equal marks.

iv. Use of non-programmable scientific calculators is allowed.

1. Attempt any 6 of the following: **[6×3=18]**

   a) An electron in an atom is placed in a magnetic field of strength $B = 1\text{T}$. Calculate the energy shift in the strong Zeeman effect for a state with $m_l = 1, m_s = -\frac{1}{2}$. Use Bohr magneton: $\mu_B = 9.27 \times 10^{-24}$J/T

   b) A particle is confined in a one-dimensional infinite potential well of width a. A weak perturbation of the form $H' = \lambda x$ is applied, where λ is a constant. Determine which unperturbed states contribute to the first-order correction to the ground-state wavefunction.

   c) A perturbation acts for time interval δt. Determine whether the process is sudden or adiabatic if $\delta t \ll \hbar/\delta E$ where δE is the characteristic energy difference of the system. Justify your answer.

   d) For a quantum transition, the matrix element is 3×10^{-24} J and the density of states is 5×10^{22} J^{-1}. Using Fermi's Golden Rule, calculate the transition rate.

   e) Under what conditions does scattering amplitude have azimuthal symmetry? Justify your answer.

   f) Show that the scattering matrix (S-matrix) is unitary and explain the physical significance of this property.

g) A beam of low-energy particles with wave number $k = 1 \times 10^{10}\,\mathrm{m}^{-1}$ is scattered by a spherically symmetric potential. Determine the differential cross-section if only the s-wave with phase shift $\delta_0 = \frac{\pi}{4}$ contributes and all higher partial wave phase shifts are negligible.

2.

a) Derive the variational principle for the ground state energy and show that for any arbitrary normalized trial wavefunction $|\Psi_t\rangle$, the expectation value of the Hamiltonian provides an upper bound to the true ground state energy $\langle\Psi_t|H|\Psi_t\rangle \geq E_0$. Under what condition does equality hold? Explain how the variational principle may be extended to estimate the first excited state energy. [9]

b) Consider a two-level quantum system with states $|a\rangle$ and $|b\rangle$ being the eigenstates of unperturbed Hamiltonion $H^{(0)}$ having energies E_a and $E_b$respectively. Define $\omega_{ab} = \frac{E_a - E_b}{\hbar}$. A time-dependent perturbation $H'(t) = V\cos(\omega t)$ is applied, where $V_{ab} = \langle a|V|b\rangle \neq 0$ and $V_{aa} = V_{bb} = 0$. Using first-order time-dependent perturbation theory, derive an expression for the transition amplitude $c_b(t)$ for the transition $|a\rangle \to |b\rangle$. Hence show that the transition probability $P_{a\to b}(t)$ is proportional to

$$\left[\frac{\sin^2\left(\frac{(\omega_{ab} - \omega)t}{2}\right)}{(\omega_{ab} - \omega)^2} + \frac{\sin^2\left(\frac{(\omega_{ab} + \omega)t}{2}\right)}{(\omega_{ab} + \omega)^2}\right].$$

[9]

3.

a) Consider the Hamiltonian

$$H = V_0 \begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}, \quad \epsilon \ll 1$$

i. Write down the eigenvalues and eigenvectors of the unperturbed Hamiltonian H^0 ($\epsilon = 0$). Identify the degenerate and non-degenerate eigenvalues.
ii. Using first-order degenerate perturbation theory, find the first order correction to the two initially degenerate eigenvalues.
iii. Use first order non-degenerate perturbation theory to find the corrected eigenvalue corresponding to the non-degenerate eigenvector of H^0.

[9]

b) Derive the expression for the differential cross-section in terms of the scattering amplitude and hence obtain the expression for the total cross-section. Discuss the physical interpretation of these quantities in terms of measurable scattering processes.

[9]

4.

a) Define the interaction-picture state vector in terms of the Schrödinger-picture state vector. Starting from the Schrödinger equation, derive the equation of motion for the interaction-picture state vector. State one advantage of the interaction picture in time-dependent perturbation theory. [9]

b) The spin–orbit interaction of an electron in a central potential is described by

$$H_{SO} = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\,\boldsymbol{L}\cdot\boldsymbol{S}.$$

Obtain the explicit radial dependence of the interaction for a Coulomb potential. Express $\boldsymbol{L}\cdot\boldsymbol{S}$ in terms of J^2, L^2, and S^2, and hence determine the eigenvalue of $\boldsymbol{L}\cdot\boldsymbol{S}$ for a state labelled by j, ℓ, s. [9]

5.

a) A particle is initially in the ground state of an infinite, one-dimensional potential well, with walls at $x = 0$ and $x = a$. If the wall of this well at $x = a$ is now suddenly moved (at $t = 0$), to $x = 8a$, calculate the probability of finding the particle in the ground state, and first excited state, of the new potential well. [9]

b) Using partial wave analysis, describe the asymptotic behaviour of the radial wavefunction for a particle scattered by a central potential. Show that the large-r form of the radial solution can be written in terms of phase shifts. Explain how the effect of the potential is entirely contained in the phase shifts. [9]

6.

a) Calculate the scattering cross-section for a low-energy particle from a potential given by

$$V(r) = \begin{cases} -V_0, & r < a \\ 0, & r > a \end{cases}$$

using Born approximation. [9]

b) Consider scattering of a particle of mass m and energy $E = \frac{\hbar^2 k^2}{2m}$ by a repulsive potential $V(r) = B\,\delta(r-a)$ where B and a are positive constants. Obtain an equation that determines s-wave phase shift $\delta_0(k)$. [9]