

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2249
Unique Paper Code : 2352203601
Name of the Paper : DSC – Probability and Statistics
Name of the Course : Bachelor of Arts / Bachelor of Science (Programme)
Semester : VI
Duration : 3 Hours
with Mathematics as non-major/minor

Your Roll No.....

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Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. All questions carry equal marks.
5. Use of non-programmable scientific calculator and statistical tables is permitted.

1. (a) Every corporation has a governing board of directors. The number of individuals on a board varies from one corporation to another. Consider the accompanying data on the number of directors on each board in a random sample of 204 corporations.

No. of directors	4	5	6	7	8	9	10	11	12	13	14	15	16	17	21	24	32
Frequency	3	12	13	25	24	42	23	19	16	11	5	4	1	3	1	1	1

Construct a histogram of the data based on relative frequency and comment on any interesting features.

- (b) Calculate the mean and variance of the following data.

116.4	115.9	114.6	115.2	115.8
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Subtract 100 from each observation to obtain a sample of transformed values. Now calculate the sample variance of these transformed values and compare it the variance of original data.

P.T.O.

- (c) The accompanying data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine:

1.6	2.0	2.6	3.0	3.5	3.9	4.5	4.6	4.8	5.0
5.1	5.3	5.4	5.6	5.8	6.0	6.0	6.1	6.3	6.5

Compute the median, upper fourth and lower fourth.

2. (a) Consider randomly selecting a student at a large university, and let A be the event that the selected student has a Visa card and B be the analogous event for MasterCard. Suppose that

$$P(A) = 0.6, P(B) = 0.4 \text{ and } P(A \cap B) = 0.3.$$

- (i) Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.

- (ii) Calculate the probability that the selected student has exactly one of the two types of cards.

- (b) States Baye's theorem.

At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2) and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested?

- (c) A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers.

3. (a) A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory. The accompanying table gives the distribution of Y = the amount of memory in a purchased drive :

y	1	2	4	8	16
P(y)	.05	.10	.35	.40	.10

- (i) Find the cdf of Y.

- (ii) Plot the graph of cdf.

- (b) Suppose $E(X) = 5$ and $E[X(X - 1)] = 27.5$. Find
- $E(X^2)$
 - $V(X)$.
- (c) A company that produces fine crystal knows from experience that 10% of its goblets have cosmetic flaws and must be classified as "seconds."
- Among six randomly selected goblets, how likely is it that only one is a second?
 - Among six randomly selected goblets, what is the probability that at least two are seconds?
4. (a) A pediatrician wishes to recruit 5 couples, each of whom is expecting their first child, to participate in a new natural childbirth regimen. Let $\bar{p} = P$ (a randomly selected couple agrees to participate). If $p = .2$, what is the probability that 15 couples must be asked before 5 are found who agree to participate? That is, with $S = \{\text{agrees to participate}\}$ and $F = \{\text{doesn't agree to participate}\}$, what is the probability that 10 F's occur before the fifth S?
- (b) Let X denote the number of traps (defects of a certain kind) in a particular type of metal oxide semiconductor transistor, and suppose it has a Poisson distribution with $\mu = 2$.
- Find the probability that there are exactly three traps.
 - Find the probability that there are at most three traps.
- (c) Let X denote the amount of time a book on two-hour reserve is actually checked out, and suppose the cdf is
- $$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$
- Obtain the probability density function $f(x)$.
 - Calculate $P(X \leq 1)$.
5. (a) Suppose the pdf of the magnitude X of a dynamic load on a bridge (in newtons) is given by

$$f(x) = \begin{cases} \frac{1}{8} + \frac{3}{8}x & 0 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that the magnitude of a dynamic load is between 0.5 to 1.5 newton. Also, find the probability that the load exceeds 1 newton.

- (b) The authors used data collected from an experiment with a specified initial crack length and number of loading cycles to propose a normal distribution with mean value 5.496 mm and standard deviation 0.067 mm for the random variable X = final crack depth. For this model, what value of final crack depth would be exceeded by only 0.5% of all cracks under these circumstances?
- (c) The breakdown voltage of a randomly chosen diode of a particular type is known to be normally distributed. What is the probability that a diode's breakdown voltage is within 1 standard deviation of its mean value? And, what is the probability that a diode's breakdown voltage is within 2 standard deviation of its mean value?
6. (a) Consider the following data on y = porosity (%) is related to x = unit weight (pcf) in concrete specimens.

x	99	101	102	103	105	107	108	110
y	29	28	27	25	23	22	21	20

Find the equation of regression line. And estimate the porosity (y) when unit weight (x) is 100.

- (b) The amount of a particular impurity in a batch of a certain chemical product is a random variable with mean value 4.0 g and standard deviation 1.5 g. If 50 batches are independently prepared, what is the (approximate) probability that the sample mean amount of impurity $\bar{X}$ is between 3.5 and 3.8 g?
- (c) Consider the following data of on x = rainfall volume (m^3) and y = runoff volume (m^3) for a particular location.

x	0.5	1.2	1.4	1.7	2.3	3.0	4.0	4.7	5.5	6.7
y	0.4	1.0	1.3	1.5	1.5	2.5	2.7	4.6	3.8	4.6

Compute the coefficient of correlation between x and y .

(4000)