[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1737 **L**

Unique Paper Code : 2372013601

Name of the Paper : Testing of Hypothesis (NEP-UGCF)

Name of the Course : **B.Sc.(H) Statistics**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory. Select **two** questions each from **Section A** and **Section B**.
3. Use of a simple scientific calculator (non-programmable) is allowed.
4. Use of statistical tables is permitted.

1. Attempt any **FIVE** parts: (5×6)

(a) Define the power function of a statistical test. What are the relationships of the power function with Type I and Type II errors?

(b) A new teaching pedagogy claims that the success rate of this pedagogy in cracking entrance exams is 90%. A competitor claims that the recovery rate is just 70%. In order to reach at a decision, the pedagogy is tested on a group of 30 students and the number of successes is observed. The claim of the pedagogy will be accepted if the number of successes is at least 21. Set the null and the alternate hypotheses and calculate α and β.

(c) If there are $n_1 = 5$ letters of one kind and $n_2 = 5$ letters of another kind, what is the probability of (a) 6 runs; (b) 7 runs?

(d) Hypertension can be controlled by diet. In order to test this claim, a diet plan was administered on 11 people, with a history of high blood pressure for three months. Their blood pressures were measured repeatedly before the plan, during the plan and after its completion. The following are the average upper blood pressures before and after the plan:

Before:	187	183	132	161	197	206	177	215	147	208	166
After:	120	126	120	130	193	201	180	203	149	195	158

Is there any evidence of the effectiveness of the plan at 5% level of significance? Use a Paired sign test to verify the claim.

(e) Differentiate between the ratio and ordinal scales of measurement giving suitable examples.

(f) A coin is tossed 14 times. The following sequence of Heads (H) and Tails (T) has been obtained:

H T T H H H T H T T H H T H

Test whether the heads and tails occur in random order at 5% level of significance.

(Critical Interval = (3,12))

(g) Explain the difference between a SPRT and a fixed sized sample test. What is the significance of the OC function of a SPRT.

Section A

2(a) Define the most powerful test. Prove that a critical region determined by the Neymann-Pearson lemma is the most powerful critical region for testing a simple null hypothesis against a simple alternative hypothesis.

(b) Let X_1, X_2 be a random sample from $f(x,\theta)=\theta x^{\theta-1};\quad 0\le x\le 1, \theta>0$. It is given that for testing $H_0:\theta=1$ against $H_1:\theta=2$ is $C=\left\{(X_1,X_2):x_1x_2\ge\frac{3}{4}\right\}$. Find the sizes of Type I and Type II errors for the test. (8, 7)

3(a) Show that the LRT for testing based on a random sample of size n from a population with density $f(x;\alpha;\beta)=\frac{1}{2\beta};\quad \alpha-\beta\le x\le\alpha+\beta$ is $\left(\frac{R}{2Z}\right)^n$ where $R=X_{(n)}-X_{(1)}$ and $Z=\max\left(-X_{(1)},X_{(n)}\right)$.

(b) Two independent observations x_1, x_2 are made on a random variable X having the density function $f(x;\theta)=\frac{1}{\theta}e^{-\frac{x}{\theta}};-\infty<x<\infty;\theta>0$. In order to test the hypothesis $H_0:\theta=2$ against $H_1:\theta=4$, the rejection region is $x_1+x_2>9.5$. Find the level of significance and the power of the test. (8,7)

4(a) For the distribution

$$dF = \begin{cases} \beta \exp\{-\beta(x-\gamma)\}\, dx, & x \geq \gamma \\ 0, & x < \gamma \end{cases}$$

show that for testing the hypothesis $H_0 : \beta = \beta_0, \gamma = \gamma_0$ against the alternative $H_1 : \beta = \beta_1, \gamma = \gamma_1$, the best critical region is given by

$$\bar{x} \leq \frac{1}{\beta_1 - \beta_0}\left\{\gamma_1\beta_1 - \gamma_0\beta_0 - \frac{1}{n}\log(k) + log\frac{\beta_1}{\beta_0}\right\}$$

provided that the admissible hypothesis is restricted by the condition $\gamma_1 \leq \gamma_0, \beta_1 \geq \beta_0$.

(b) Define Monotone Likelihood Ratio (MLR). Show that for the family of distributions $\{N(\theta,\sigma^2); \theta \in \mathbb{R}\}$, there exists an MLR. (8,7)

Section B

5(a) Define ASN function of SPRT. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a normal distribution with mean μ and variance 100. SPRT was done to test the hypothesis $H_0 : \mu = 50$ against $H_0 : \mu = 65$ for given $\alpha = 0.05$ *and* $\beta = 0.1$. Find ASN

(i) when the null hypothesis is true

(ii) when the alternative hypothesis is true

(b) Suppose that the random variable X follows Bernoulli distribution with parameter θ. Develop a SPRT for testing

$H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$.

Also obtain its OC and ASN functions.

(8, 7)

6(a) For the SPRT of strength (α, β) and for $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$, prove that

(i) $\alpha' \leq \frac{\alpha}{1-\beta}$, $\beta' \leq \frac{\beta}{1-\alpha}$

(ii) $\alpha' + \beta' \leq \alpha + \beta$, where (α', β') are the resulting strengths of the test.

(b) Explain the non-parametric median test for testing that the two samples come from the populations having the same median. (8,7)

7(a) Explain the Kruskal-Wallis test for testing the null hypothesis that *K*- samples come from same continuous population. Explain the large sample behaviour of the test.

(b) In order to compare three bowling balls, a professional bowler bowls five games with each of the three balls. The following are the results of the five games:

Ball 1:	208	220	247	192	229
Ball 2:	216	196	189	205	210
Ball 3:	226	218	252	225	202

Write the distribution of the test statistic. Can it be concluded at 5% level of significance that the same scores can be expected from each of the three balls? (tabulated value = 5.78).

(8,7)