[This question paper contains 4 printed pages.]

Roll No. ________

Sr. No. of questions Paper : **1735**

Name of the Department: Physics and Astrophysics

Name of Course : B.Sc. Hons.–(Physics)_NEP: UGCF-2022

Name of the Paper: Statistical Mechanics

Semester : VI

Unique Paper Code : 2222013601

Question paper Set No. : II

Duration: 3 Hours   Maximum Marks: 90

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt **five** questions in all

Question No. 1 is compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. Attempt any six. (6 x 3 = 18)

i) At a high temperature T, calculate the average energy per particle of a non-interacting gas of two-dimensional classical harmonic oscillators (No derivation required)

ii) 5 distinguishable particles are to be distributed amongst three energy levels with energy 0, ε and 2ε. For a fixed energy of 5ε, enumerate the number of macrostates. Identify which macrostate is the most probable?

iii) Consider the blackbody radiation contained in a cavity whose walls are at temperature T = 500 K. The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to 1000 K and the radiation is allowed to come to equilibrium at the new temperature, by what factor will the entropy change for the blackbody radiation in the cavity.

iv) Calculate the chemical potential of strongly degenerate system of N bosons (N >> 1) at temperature T ($T < T_C$), if the energy of the lowest state is ε_0.

v) Explain and draw a qualitative graph of the temperature dependence of pressure exerted by bosons for all temperatures starting from T = 0 including $T = T_c$ and going upto $T >> T_C$.

vi) Show graphically the variation of radius of white dwarf star (consisting of relativistic electrons) as a function of its mass. Hence explain the physical significance of Chandrasekhar mass limit.

vii) Calculate the value of $\frac{\partial f(\varepsilon)}{\partial \varepsilon}$ at $\varepsilon = \mu$, where $f(\varepsilon)$ represents Fermi-Dirac distribution function. Explain the result.

2(a)Consider an isolated system of $N \gg 1$ weakly interacting classical particles. These particles can be in any of the states with energies $0, \varepsilon, 2\varepsilon, 3\varepsilon, \ldots$.The temperature of the system may be written as

$$\frac{1}{T} \cong \frac{\Delta S}{\Delta E}$$

where ΔS is the change in the entropy of the system corresponding to change in internal energy of the system by ΔE.

i) What is the entropy of the system when it is in its ground state.

ii) If the total energy of the system is ε, what will be its entropy?

iii) Determine the change in entropy and the corresponding temperature when the energy of the system is increased from ε to 2ε ? (12)

(b) An isolated physical system consisted of N distinguishable particles is characterized by two energy states: a ground state of energy ε_1 with a degeneracy g_1 and number of particles n_1. An excited state is characterised by energy ε_2 with degeneracy g_2 having number of particles n_2. Show that the entropy of this system can be written as

$$S = -k_B[P_1 ln(P_1/g_1) + P_2 ln(P_2/g_2)]$$

where P_1 and P_2 are the probabilities that the system will be found in the ground and the first excited states respectively (Here $N = n_1 + n_2$). (6)

3 (a) A system of H-atoms is enclosed in a cavity of volume V at a very high temperature that results in the thermal ionization of the H atom into H^+ ion and e^- electron.

$$H \rightleftharpoons H^+ + e^-$$

Show that the degree of ionization x for this system in terms of the temperature T., pressure p and ionization energy ε^* can be expressed as:

$$\frac{x^2}{1-2x} = \left(\frac{2\pi m_e}{h^2}\right)^{3/2} \frac{(k_B T)^{5/2}}{p} \left(\frac{2g_I}{g_a}\right) \exp\left(-\frac{\varepsilon^*}{k_B T}\right)$$

where g_I, g_a are the ground state degeneracies of the ion and the atom respectively and m_e is the mass of electron. (12)

(b) A system of N classical particles confined to a volume V has the partition function given by

$$Z_N = (V - Nb)^N (2\pi m k_B T)^{3N/2} \exp(aN^2/V k_B T)$$

Obtain the expressions for pressure, total internal energy and the specific heat at constant volume for this system. (6)

4 (a) (i) What assumptions were made by Planck to derive the black body radiation formula.

(ii) The number of vibration modes per unit volume in frequency interval ν to $\nu + d\nu$ is $\frac{8\pi\nu^2}{C^3} d\nu$. Use this to derive the black body radiation formula as proposed by Planck.

(iii) Derive Wien's displacement law in the form involving peak frequency and temperature, $\nu_{peak} = 2.822 \frac{k_B}{h} T$. (Use $3(e^x - 1) - xe^x = 0 \rightarrow x = 2.822$)

(12)

(b) The power radiated by a blackbody at temperature T_1 is P_1 and it radiates the maximum energy at λ_1. If the temperature of the same blackbody is changed to T_2 so that it radiates the maximum energy at $\lambda_2 = \lambda_1/2$. The power radiated by it becomes P_2. Find the ratio of P_2 / P_1. (6)

5(a)Obtain an expression for thermodynamic probability of a macrostate using Fermi-Dirac statistics, hence derive the equilibrium distribution function for a system of fermions having fixed number of particles and energy at temperature T using Stirling's approximation. Also calculate the value of unknown Lagrange multipliers. (12)

(b) Consider a system of N electrons moving non relativistically in a two-dimensional sheet of area A at temperature T = 0 °K. Calculate the Fermi energy of this system. (6)

6. Consider a gas of N non-interacting Bose particles with spin 0 and mass m in Volume V. In the ultra-relativistic limit, the dispersion relation (energy-momentum relation) can be approximated as: $\varepsilon = c\,p$, where c is a constant and p is the momentum.

(a)Prove that the condensation temperature, $T_C = \frac{hc}{2\pi k} \left[\frac{\pi^2 n}{\zeta(3)}\right]^{1/3}$, where $n = N/V$.

(12)

(b)Show that the number of particles in the ground state (N_0) is equal to

$$N_0 = N \left[1 - \left(\frac{T}{T_C}\right)^3\right]$$ (6)

Useful Formulae and Constants

Stefan's constant $\sigma = 5.67 \times 10^{-8}\, Wm^{-2}K^{-4}$

Boltzmann constant $k_B = 1.38 \times 10^{-23} JK^{-1}$

Planck's constant $h = 6.62 \times 10^{-34} Js$

Gas constant $R = 8.314 J K^{-1} mol^{-1}$

$\int_0^\infty x^3 e^{-x} dx = 6$; $\int_{-\infty}^{\infty} exp(-ax^2)\, dx = \sqrt{\frac{\pi}{a}}$; $\int_{-\infty}^{\infty} exp(-a\, x^4)\, dx = \frac{\Gamma(1/4)}{2\, a^{1/4}}$