[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 101 **L**

Unique Paper Code : 2352204801

Name of the Paper : Topics in Multivariate Calculus

Name of the Course : **B.A./B.Sc.(Programme)**

Semester : VIII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carries equal marks.
4. Use of Calculator is not allowed.

Q.1.(a) Let f be the function defined by $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$. Is f continuous at $(0,0)$? Explain.

(b) Let $z = 4x - y^2$, where $x = u v^2$ and $y = u^3 v$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

(c) Find all second order partial derivatives for $f(x,y) = 5x^2 - 2xy + 3y^3$.

Q.2.(a) Find an equation for the tangent plane to the surface $z = \tan^{-1}(\frac{y}{x})$ at the point $P(1, \sqrt{3}, \pi/3)$.

(b) Find all relative extrema and saddle points of the function

$$f(x,y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(c) Use the method of Lagrange multipliers to find the extrema of $f(x,y) = 1 - x^2 - y^2$ subject to the constraint $x + y = 1$ with $x \geq 0$ and $y \geq 0$.

Q.3. (a) Find the volume of the solid bounded below by the rectangle R in the xy-plane and above by the graph of $z = 2x + 3y$; $R: 0 \leq x \leq 1,\ 0 \leq y \leq 2$.

(b) Sketch the region of integration and evaluate $\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$ by reversing the order of integration.

(c) Evaluate $\iint_D \frac{1}{x}\, dA$ where D is the region that lies inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1 + \cos\theta$.

Q.4. (a) Find the volume of the tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0, y = 0$ and $z = 0$

(b) Evaluate the integral $I = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} z\,dz\,dy\,dx$ using the given rectangular form, or by transforming to either cylindrical or spherical coordinates.

(c) Evaluate $\iint_D (x-y)^5\,(x+y)^3\,dy\,dx$ where D is the region in $x\,y$- plane that is bounded by the coordinate axes and the line $x + y = 1$.

Q.5 (a) Evaluate the line integral $\int_C (y\,dx - z\,dy + x\,dz)$ where C is the curve with parametric equations: $x = t^2$, $y = e^{-t}$, $z = e^t$ for $0 \le t \le 1$.

(b) Verify the vector field: $F(x,y) = (y - x^2)\,\hat{\imath} + (x + y^2)\,\hat{\jmath}$ is conservative and find a scalar potential f for F. Then evaluate the line integral: $\int_C F \cdot dR$, where C is any smooth path connecting A(0, 0) to $B(1, 1)$.

(c) Verify Green's theorem for the line integral: $\oint_C (x^2 - y^2)\,dx + 2\,x\,y\,dy$, where C is the boundary of rectangle: $0 \le x \le 1, 0 \le y \le 1$.

Q.6 (a) Evaluate: $\iint_S (x^2 + y^2)\,dS$, where S is the surface of the hemisphere: $z = \sqrt{1 - x^2 - y^2}$.

(b) Use Stokes' theorem to evaluate: $\oint_C (y\,dx - 2\,x\,dy + z\,dz)$, where C is the intersection of the surface: $z = x^2 + y^2$ and the plane: $x + y + z = 1$, oriented counter-clockwise when viewed from the origin.

(c) Use the divergence theorem to evaluate: $\iint_S F \cdot N\,dS$, where $F = x\,y\,\hat{\imath} + x\,y\,z\,\hat{\jmath} + 3\,x\,\hat{k}$ and S is the surface of the box: $0 \le x \le 1,\ 0 \le y \le 2,\ 0 \le z \le 3$ with outward unit normal vector N.