[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : **9546** **K**

Unique Paper Code : 2353200015

Name of the Paper : DSE- INTEGRAL TRANSFORMS

Name of the Course : **NEP-UGCF 2022 BACHELOR OF ARTS / BACHELOR OF SCIENCE (PROGRAMME)**

Semester : VII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all six** questions.
3. **All** questions carry equal marks.

1. Let $f(x)$ be piecewise continuous and periodic function with period 2π. The function $f(x)$ represented by a Fourier series. Hence, prove the following

$$\int_{-\pi}^{\pi} f^2(x)dx > \frac{\pi}{2}\left(a_0^2 + 2\sum_{k=1}^{n}(a_k^2 + b_k^2)\right)$$

wherea_k, b_k are Fourier coefficients, for all values of n.

OR

Find the Fourier series expansion of the periodic function $f(x) = x, \ -\pi < x < \pi$ and $f(x) = f(x + 2m\pi), \ m = 1,2\cdots$. Hence, prove

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

2. Find the maximum value of the Fourier series expansion of the following function,

$$f(x) = \begin{cases} -1, & -a < x < 0 \\ 1, & 0 \le x < a \\ 0, & otherwise. \end{cases}$$

OR

If $f(x) = e^{-x}, \ 0 < x < \infty$. Show that the Fourier cosine integral representation is $f(x) = \frac{2}{\pi}\int_0^{\infty} \frac{k \sin kx}{1+k^2} dk$.

3. Find the Fourier transform of $e^{-a|x|}$ and explain the same by plotting a graph.

OR

Find the Fourier transform of e^{-ax^2} and by using this result calculate the value of $\int_{-\infty}^{\infty} x^2 e^{-ax^2}\, dx$.

4. If $f(t) = J_0(at)$ is a Bessel function of order zero then show that

$$\mathcal{L}\{J_0(at)\} = \frac{1}{\sqrt{s^2 + a^2}}$$

OR

Obtain the convolutions of :

(a) $t * e^{at}$

(b) $\sin at * \sin at$

(c) $\cos t * e^{2t}$

(d) $t * t * t$

5. Using the Fourier transform solve the one dimensional wave equation

$$u_{tt} = c^2 u_{xx}, \quad -\infty < x < \infty, \ t > 0,$$

$$u(x,0) = 0, \ u_t(x,0) = \delta(x), \ -\infty < x < \infty$$

OR

using the Laplace transform solve the matrix differential system

$$\frac{d}{dt}x(t) = Ax(t), x(0) = \begin{pmatrix} x_0 \\ 0 \end{pmatrix}$$

where $A = \begin{pmatrix} -3 & -2 \\ 3 & 2 \end{pmatrix}$

6. A rod with diffusion constant κ contains a fuel which produces neutrons by fission. The ends of the rod are perfectly reflecting. If the initial neutron distribution is $f(x)$, find the neutron distribution $u(x,t)$ at any subsequent time t. The problem is governed by

$$u_t = ku_{xx} + bu,$$

$$u(x,0) = f(x), \quad 0 < x < l, \ t > 0,$$

$$u_x(0,t) = 0, \ u_x(l,t) = 0.$$

Using the Fourier transform solve the PDE

OR

using the Laplace transform solve the partial differential equation

$$u_t + xu_x = x$$

with initial and boundary conditions $u(x,0) = 0$ for $x > 0$, $u(0,t) = 0$ for $t > 0$.