basis B. Derive the conditions to ensure that the current b.f.s can be improved to get another b.f.s with improved objective value using simplex algorithm.

(c) Solve the following linear fractional programming problem by the simplex algorithm

$$\text{Maximize} \quad \frac{x_1}{x_1 + x_2 + 1}$$

$$\text{subject to} \quad x_1 + x_2 \le 1$$

$$x_1 + 2x_2 \le 1$$

$$x_1, x_2 \ge 0.$$

By normalizing nmr or dnr?

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 9542 **K**

Unique Paper Code : 2353010020

Name of the Paper : Optimization

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** pans from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculators is permitted.

1. (a) (i) Examine the convexity of the function

$$f(x_1, x_2) = 2x_1^2 + x_2^2 + 4x_1x_2.$$

(ii) Over what subset of $\{x: x > 0\}$ is the function $f: R \to R$ defined by $f(x) = e^{\{-ax^b\}}$ convex, where $a > 0$ and $b \geq 1$?

(b) Let $f_1, f_2, \ldots, f_k: R^n \to R$ be convex functions. Consider the function f defined by $f(x) = \sum_{j=1}^{k} \alpha_j f_j(x)$, where $\alpha_j > 0$ for j = 1,2,..., k. Show that f is convex.

(c) Let S be a nonempty convex set in R^n and let $f: S \to R$. Prove that f is convex if and only if its epigraph is a convex set.

2. (a) Consider the problem; minimize f(x) subject to $x \in S$, where $f: R^n \to R$. Define the terms global optimal, local optimal, strict local and strong local

subject to $x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0.$

(c) Starting from the point (0,0), perform two iterations of the steepest descent method to minimize $f(x_1,x_2) = 3x_1^2 - 4x_1x_2 + 2x_2^2 + 4x_1 + 6$ over $(x_1, x_2) \in R^2$. Is the obtained solution after two iterations optimal?

6. (a) Use Newton's method to minimize $f(x_1, x_2) = 8x_1^2 - 4x_1x_2 + 5x_2^2$, $(x_1, x_2) \in R^2$ starting with (6, 3).

(b) Consider the linear fractional programming problem

$$\text{Maximize } \frac{c^T x + \alpha}{d^T x + \beta}$$

subject to $Ax = b$

$x \geq 0.$

Where $x, c, d \in R^n$, $A \in R^{m \times n}$, $b \in R^m$, $\alpha, \beta \in R$. Let x_B be a basic feasible solution (b.f.s) with objective value $\theta(u, v)$.

Prove that $f(x) \geq \theta(u, v)$.

5. (a) Consider the following Quadratic Programming Problem (QPP) :

Minimize $(x_1 - x_2)^2 + x_2$

subject to $-x_1 + x_2 \leq 0$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

Express the objective in the standard QPP form $c^T x + x^T D x$. Is D positive definite? Write the KKT conditions for this problem.

(b) Use Wolfe's Method to solve the following Quadratic Programming Problem :

Maximize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

optimal solutions. Illustrate each with an example.

(b) Define a quasiconvex function. Show that every convex function is quasiconvex. Give an example of a quasiconvex function which is not convex.

(c) Define pseudoconvex functions. Is the function $f(x) = x^3$ pseudoconvex? Justify your answer.

3. (a) Suppose that $f: R^n \to R$ is differentiable at $\bar{x}$. If there is a vector d such that $\nabla f(\bar{x})^t d < 0$, then there exists a $\delta > 0$ such that $f(\bar{x} + \lambda d) < f(\bar{x})$ for each $\lambda \in (0, \delta)$, so that d is a descent direction of f at $\bar{x}$. Further show that if $\bar{x}$ is a local minimum then $\nabla f(\bar{x}) = 0$.

(b) Consider the univariate function $f(x) = (x^2 - 1)^3$. Find all local minima, maxima and points of inflection, if any.

(c) Consider the problem :

Minimize $(x_1 - 3)^2 + (x_2 - 2)^2$

Subject to

$$x_1^2 + x_2^2 \leq 5$$

$$x_1 + 2x_2 \leq 4$$

$$-x_1 \leq 0$$

$$-x_2 \leq 0.$$

Verify that the Fritz John conditions hold true at the optimal point (2,1).

4. (a) Consider the following unconstrained problem:

Minimize $2x_1^2 - x_1x_2 + x_2^2 - 3x_1 + e^{2x_1+x_2}$

Write the first order necessary optimality conditions. Is this condition also sufficient for optimality? Justify.

(b) Write the Lagrangian dual of the primal problem :

Minimize $x_1^2 + x_2^2$

Subject to $x_1^2 + x_2^2 \leq 5$

$$x_1 + 2x_2 = 4$$

$$x_1, x_2 \geq 0$$

(c) Let x be a feasible solution to the problem (P) defined as :

Minimize f(x)

Subject to $g_i(x) \leq 0$, for i = 1,2, ...m

$h_i(x) = 0$, for i = 1,2,.... l

$x \in X$

Where X is a nonempty open set in R^n; f: $R^n \rightarrow R$; g_i: $R^n \rightarrow R$ for i = 1,2,..... m; h_i: $R^n \rightarrow R$ for i = 1,2,... l.

Let (u,v) be a feasible solution to its dual (D) with