(c) Define a Hamming binary code Ham $(r, 2)$. Let the Parity check matrix of binary Hamming code Ham(3,2) is

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Decode the received vector 1101011.

(7.5)



[This question paper contains 8 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 9541** **K**

Unique Paper Code : 2353010019

Name of the Paper : Information Theory and Coding

Name of the Course : **B.Sc. (H) Mathematics (UGCF)**

Semester : VII – DSE

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

P.T.O.

1. (a) Define entropy of a binary source. Calculate the entropy of a source with an alphabet having 64 symbols, where each symbol is equally probable. (7.5)

(b) Verify the additive rule of entropies for the probability scheme [A,M = B∪C], where the input probabilities are $P(A)=\frac{1}{5}$, $P(B)=\frac{4}{15}$, $P(C)=\frac{8}{15}$. (7.5)

(c) A transmitter has an alphabet consisting of five letters $\{x_1, x_2, x_3, x_4, x_5\}$ and the receiver has an alphabet of four letters $\{y_1, y_2, y_3, y_4\}$. The joint probabilities are given as :

$$\begin{array}{c|cccc} & y_1 & y_2 & y_3 & y_4 \\ \hline x_1 & 0.25 & 0 & 0 & 0 \\ x_2 & .15 & .25 & 0 & 0 \\ x_3 & 0 & 0.05 & 0.10 & 0 \\ x_4 & 0 & 0 & 0.05 & 0.10 \\ x_5 & 0 & 0 & 0.05 & 0 \end{array}$$

Compute all the entropies for this channel. (7.5)



6. (a) Construct the standard array for the code having the generator matrix

$$G=\begin{bmatrix}1 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 1 & 1\end{bmatrix}.$$

Using this array decode the received vector 01111 and 01011. (7.5)

(b) Let C be a ternary linear code with generator matrix

$$G=\begin{bmatrix}1 & 1 & 1 & 0\\ 2 & 0 & 1 & 1\end{bmatrix}.$$

Then

(i) Find a parity check matrix for C in the standard form.

(ii) Use syndrome decoding to decode the received vectors 2121, 1201 and 2222. (7.5)

5. (a) Define a generator matrix of a linear code. Prove that every codeword of a code is uniquely expressible as a linear combination of rows of the gene rator matrix of the code. (7.5)

(b) Prove that a code having minimum distance 2t + 1 can correct t or less errors. (7.5)

(c) Let C be a linear code over GF(2) having the generator matrix

$$G=\begin{bmatrix}1&0&0&1&1&0&1\\0&1&0&1&0&1&1\\0&0&1&0&1&1&1\end{bmatrix}.$$

Then find the all code vectors of C. (7.5)



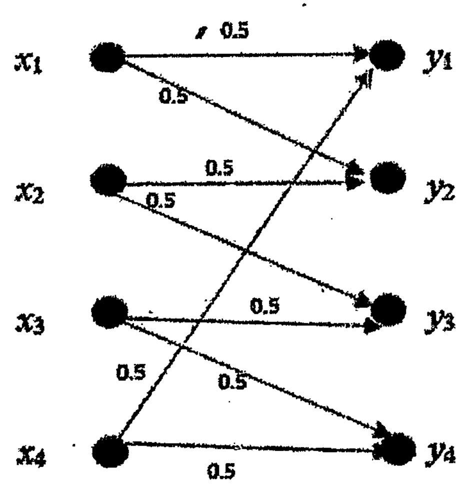
2. (a) Show that logarithmic function is convex upward function and hence prove that $H(X) \le \log(m)$ where m is the number of source points. (7.5)

(b) Show that for the joint random variable (X, Y) and the mutual information I(X, Y)

(i) $I(X;Y) = H(Y) - H(Y|X)$

(ii) $I(X;Y) = H(X) + H(Y) - H(XY)$. (7.5)

(c) Given below the noise characteristic



For equi-probable transmission probabilities of the messages $x_1$, $x_2$, $x_3$, $x_4$ determine the following:

(i) Receiver entropy H(Y)

(ii) Conditional entropy H(Y|X)

(iii) Mutual in formation I(X;Y) (7.5)

3. (a) Define channel capacity, redundancy and efficiency of a noise free channel. The joint probability matrix of a channel with binary input and output is given below:

$$\begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}.$$

Find the different entropies and the mutual information. (7.5)

(b) State and prove the Log sum inequality. (7.5)



(c) A codeword of the code {10010, 01010} is transmitted through a BSC with crossover probability p = 0.2, and a nearest-codeword decoder D is applied to the received word.

Compute the decoding error probability $P_{err}$ of D. (7.5)

4. (a) Define $A_q(n, d)$ and show that $A_2(5,3) = 4$. (7.5)

(b) Listed below are the blocks of a (7, 7, 3, 3, 1) - design. Use this to construct a binary (7, 16, 3) – code.

{1, 2, 4}, {2, 3, 5}, {3, 4, 6}, {4, 5, 7}, {5, 6, 1}, {6, 7, 2}, {7, 1, 3}. (7.5)

(c) Show that every square integer is congruent (mod 4) to either 0 or 1. (7.5)