[This question paper contains 2 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper : 9539

Unique Paper Code : 2353010017

Name of the Paper : Dynamical Systems

Name of the Course : **B.Sc. (H) Mathematics – DSE**

Semester : VII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Each question has **three** parts and each part is of **7.5** marks. Attempt any **two** parts from each question.

1. a) Let $Q(x) = x^2 - 2$. Apply Newton's method with the initial guess $x_0 = 1$. Compute x_1, x_2, and x_3. Compare these values with the exact root $\sqrt{2}$.

   b) Define forward asymptotic points for a periodic function. Discuss the forward asymptotic points of $f(x) = \frac{1}{2}(x + \frac{2}{x})$

   c) Define $g(\theta) = \theta + \epsilon \sin(2\theta)$ for $0 < \epsilon < \frac{1}{2}$.

      i) Find the periodic points of the function g.

      ii) Sketch the phase portraits of g.

2. a) Let p be a hyperbolic fixed point with $|f'(p)| > 1$. Then there is an open interval U of p such that, if $x \in U, x \neq p$, then there exists $k>0$ such that $f^k(x) \notin U$.

   b) Find all the periodic points of $f(x) = x - x^2$. Classify them as attracting, repelling with the phase portraits.

   c) For the function $F_2(x) = 2x(1-x)$. Prove that $F_2^n(x) \to 1/2$ as $n \to \infty$, whenever $0 < x < 1$.

3. a) i) Let $s, t \in \Sigma_2$ and suppose $s_i = t_i$ for $i \leq n$, *then prove that* $d[s,t] \leq \frac{1}{2^n}$ where $d[s,t] = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^i}$. Conversely, if $d[s,t] < \frac{1}{2^n}$, then $s_i = t_i$ for $i \leq n$.

      ii) Compute $d[s,t]$ for s = (1000 1000 1000...) and t = (101010101010...).

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b) Define $\sigma: \sum_2 \to \sum_2$ as $\sigma(s_0 s_1 s_2 \ldots) = (s_1 s_2 s_3 \ldots)$. Prove that the cardinality of $Per_n(\sigma) = 2^n$.

c) Let $Q_c(x) = x^2 + c$. Prove that if $c < 1/4$,there is a unique $\mu > 1$ such that Q_c is topologically conjugate to $F_\mu(x) = \mu x(1-x)$ via a map of the form $h(x) = \alpha x + \beta$.

4. a) Define Periodic point. Find a collection $\mathcal{F}$ of blocks over $\{0, 1\}$ so that $X_{\mathcal{F}} = \Phi$.

b) Let X be the full $\mathcal{A}$-shift. Explain statements that no longer hold if we merely assume that X is a shift space.

c) Define the higher block shift. Prove that the higher block shifts of a shift space are also shift spaces.

5. a) The image of a shift space under a sliding block code is a shift space.

b) Define a shift of finite type with example. Prove that if X is a shift of finite type, then there is an $M \geq 0$ such that X is M- step.

c) Suppose that H is a subgraph of G. What is the relationship between the adjacency matrices A_H and A_G?

6. a) Let X be a shift space. If X^N is shift of finite type, must X be a shift of finite type?

b) Define Pseudo orbit tracing property (Shadowing property).

c) Let f be a linear map of the Euclidean space $\mathbb{R}^n$ and d be the Euclidean metric for $\mathbb{R}^n$. Then f has Pseudo orbit tracing property (POTP) under d if and only if f is hyperbolic, i.e. it has no eigenvalues of modulus 1.