Player B

$$\text{Player A} \begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix}$$

(b) Convert the following Game Problem into a Linear Programming Problem for player A and player B and solve it by Simplex Method :

Player B

$$\text{Player A} \begin{bmatrix} 1 & 4 & 2 \\ 5 & 3 & 6 \end{bmatrix}.$$

(c) Using the concept of dominance solve the game whose pay-off matrix for the players A and B is given as :

Player B

$$\text{Player A} \begin{bmatrix} -5 & -1 & -1 \\ 4 & 0 & 2 \\ 5 & 2 & 3 \end{bmatrix}$$



[This question paper contains 8 printed pages.]

**Your Roll No.....**

**Sr. No. of Question Paper : 6568** **K**

Unique Paper Code : 2352203502

Name of the Paper : DSC-5 : LINEAR PROGRAMMING

Name of the Course : **B.A. (Prog.) Mathematics with Major**

Semester : V

Duration : 3 Hours Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) Define a convex set and examine the convexity of the set

$$S = \{(x_1,x_2) : x_2^2 \leq 4x_1\}.$$

(b) Solve the following LPP graphically

Maximize $z = 4x_1 + 3x_2$

Subject to

$$x_1 + x_2 \leq 50$$
$$x_1 + 2x_2 \leq 80$$
$$2x_1 + x_2 \geq 20$$
$$x_1, x_2 \geq 0$$

(c) Find all the basic solutions for

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5.$$

2. (a) Solve the following linear programming problem by Simplex method

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 \leq 4$

$$x_2 \leq 6$$
$$x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0.$$



How should cars be assigned to customers so as to minimize the distance travelled? Also, find the minimum distance.

(c) Six machines $M_1$, $M_2$, $M_3$, $M_4$, $M_5$ and $M_6$ are to be located in six places $P_1$, $P_2$, $P_3$, P4, $P_5$ and $P_6$. $C_{ij}$ the cost of locating machine $M_i$ at place $P_j$ is given in the matrix below:

| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ |
|---|---|---|---|---|---|---|
| $M_1$ | 20 | 23 | 18 | 10 | 16 | 20 |
| $M_2$ | 50 | 20 | 17 | 16 | 15 | 11 |
| $M_3$ | 60 | 30 | 40 | 55 | 8 | 7 |
| $M_4$ | 6 | 7 | 10 | 20 | 25 | 9 |
| $M_5$ | 18 | 19 | 28 | 17 | 60 | 70 |
| $M_6$ | 9 | 10 | 20 | 30 | 40 | 55 |

Determine the optimum assignment schedule and find the optimal cost.

6. (a) Find the best strategy for each player, and the value of a play of the game A and B. The pay-off matrix for the game is :

| Persons \ Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | 8 | 4 | 2 | 6 | 1 |
| B | 0 | 9 | 5 | 5 | 4 |
| C | 3 | 8 | 9 | 2 | 6 |
| D | 4 | 3 | 1 | 0 | 3 |
| E | 9 | 5 | 8 | 9 | 5 |

Determine the optimum assignment schedule and find the optimal cost.

(b) A car hiring company has one car at each of six depots a, b, c, d, e and f. A customer requires a car in each town namely A, B, C, D, E and F. Distance (in kms) between depots (origins) and towns (destinations) are given in the following distance matrix:

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| A | 9 | 22 | 58 | 11 | 19 | 27 |
| B | 43 | 78 | 72 | 50 | 63 | 48 |
| C | 41 | 28 | 91 | 37 | 45 | 33 |
| D | 74 | 42 | 27 | 49 | 39 | 32 |
| E | 36 | 11 | 57 | 22 | 25 | 18 |
| F | 3 | 56 | 53 | 31 | 17 | 28 |



(b) Solve the following linear programming problem by Simplex method

Maximize $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0.$

(c) Solve the following linear programming problem by Simplex method

Maximize $z = 2x_1 + x_2 + 3x_3$

Subject to $x_1 + x_2 + 2x_3 \leq 5$

$2x_1 + 3x_2 + 4x_3 = 12$

$x_1, x_2, x_3 \geq 0.$

3. (a) Solve the following LPP graphically

Maximize $z = 3x_1 + 2x_2$

Subject to

$x_1 - x_2 \leq 1$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0.$

P.T.O.

(b) Find the dual of the given primal problem:

Minimize $x_1 + 2x_2 - 3x_3$

subject to

$$x_1 + 2x_2 - x_3 + x_4 \geq 6$$
$$2x_1 - 3x_3 \leq 8$$
$$x_2 + x_3 + x_4 = 10$$
$$x_1, x_2 \geq 0, x_3 \leq 0, x_4 \geq 0.$$

(c) Write the dual for the following linear problem :

Maximize $-2x_2 + x_3 - 3x_4$

subject to

$$x_1 + x_2 - x_4 = 5$$
$$2x_2 + 3x_3 - 4x_4 \geq -3$$
$$3x_1 - x_2 + x_4 \leq 1$$
$$x_1, x_2, x_3 \geq 0, x_4 \leq 0.$$

4. (a) Find the initial basic feasible solution using North-west Comer Rule, Least Cost Method and Vogel's approximation method. Compare all the three solutions.

| | | *Warehouses* | | | | |
|---|---|---|---|---|---|---|
| | | $W_1$ | $W_2$ | $W_3$ | $W_4$ | *Supply* |
| *Factories* | $F_1$ | 48 | 60 | 56 | 58 | 140 |
| | $F_2$ | 45 | 55 | 53 | 60 | 260 |
| | $F_3$ | 50 | 65 | 60 | 62 | 360 |
| | $F_4$ | 52 | 64 | 55 | 61 | 220 |
| | *Demand* | 200 | 320 | 250 | 210 | |



(b) Starting with initial basic feasible solution using Least Cost Method, find the optimal solution of the following transportation problem

| | | *Destination* | | | | |
|---|---|---|---|---|---|---|
| | | *1* | *2* | *3* | *4* | *Supply* |
| *Source* | *1* | 6 | 1 | 9 | 3 | 70 |
| | *2* | 11 | 5 | 2 | 8 | 55 |
| | *3* | 10 | 12 | 4 | 7 | 90 |
| | *Demand* | 85 | 35 | 50 | 45 | |

(c) Obtain the initial basic feasible solution to the following transportation problem using Vogel's approximation method. Is this solution an optimal solution? If not, obtain the optimal solution.

| | | *Warehouses* | | | | |
|---|---|---|---|---|---|---|
| | | $W_1$ | $W_2$ | $W_3$ | $W_4$ | *Supply* |
| *Factories* | $F_1$ | 19 | 30 | 50 | 10 | 7 |
| | $F_2$ | 70 | 30 | 40 | 60 | 9 |
| | $F_3$ | 40 | 8 | 70 | 20 | 18 |
| | *Demand* | 5 | 8 | 7 | 14 | |

5. (a) Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows: