(c) Evaluate $\lim_{x \to 0} \tan\left(\frac{\pi(\tan x - x)}{x^2 \tan x}\right)$.

5. (a) Find all the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + 1 = 0.$$

(b) Find the nature of the origin on the curve

$$a^4y^2 = x^4(x^2 - a^2).$$

(c) If $u_n = \int_0^{\frac{\pi}{2}} \cos^n(x)\, dx$, show that $u_n = \left(\frac{n-1}{n}\right) u_{n-2}$.

Hence, evaluate u_5.

6. (a) Determine the position and character of the double points on the curve

$$(x - 2)^2 = y(y - 1)^2.$$

(b) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^m(x) \cos^n(x) dx$, where m and n are even positive integers. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^2(x) \cos^3(x) dx$.

(c) Trace the curve $y = x^3 - 3ax^2$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6530 **K**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Discuss the continuity of the function $f: \mathbb{R} \to \mathbb{R}$, defined by

$$f(x) = |x| + 1|$$

at $x = 0$ and $x = 1$.

(b) Find the n^{th} derivative of

$$y(x) = \frac{x+1}{(x-1)^2}$$

(c) If

$$u(x,y,z) = \log(x^3 + y^3 + z^3 - 3xyz).$$

then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$

2. (a) Examine the continuity and differentiability of the function $f: \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0; \\ 0, & x = 0 \end{cases}$$

at $x = 0$.

(b) If

$$y(x) = a\cos(\log x) + b\sin(\log x),$$

where a and b are constants, then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

(c) Define a homogenous function of degree n. Further, prove that if

$$u(x,y) = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right).$$

then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot(u) = 0$$

3. (a) State and prove Lagrange's Mean Value Theorem. Give its geometrical interpretation.

(b) Show that for $x > 0$,

$$\frac{x}{1+x} < \log(1+x) < x$$

(c) Evaluate

(i) $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$

(ii) $\lim_{x\to 0}\left(\frac{1}{e^x-1} - \frac{1}{x}\right)$

(iii) $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$

4. (a) Verify Rolle's Theorem for

(i) $x^3 - 6x^2 + 11x - 6$, $x \in [1,3]$

(ii) $\sin(x)$, $x \in [0,\pi]$

(b) Using Maclaurin's series, expand $f(x) = \sin x$ and $g(x) = \cos x$.