(c) Determine the solution of wave equation of semi-infinite string with free end for non- homogeneous boundary value problem:

$$u_{tt} - 16u_{xx} = 0, \quad x > 0, \quad t > 0,$$
$$u(x,0) = \cos x, u_t(x,0) = 2, \quad x \geq 0,$$
$$u_x(0,t) = 1 - t, \ t \geq 0.$$

6. (a) Obtain the solution for a semi-infinite string with free end:

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \infty, \quad t > 0,$$
$$u(x,0) = f(x), \quad 0 \leq x < \infty,$$
$$u_t(x,0) = g(x), \quad 0 \leq x < \infty,$$
$$u_x(0,t) = 0, \quad 0 \leq t < \infty.$$

(b) Determine the solution of wave equation for non-homogeneous boundary value problem of semi-infinite string with fixed end:

$$u_{tt} - 4u_{xx} = 0, \quad x > 0, \quad t > 0,$$
$$u(x,0) = \log(1+x), \ x \geq 0$$
$$u_t(x,0) = 1, \ x \geq 0,$$
$$u(0,t) = t, \quad t \geq 0.$$

(c) Determine the solution of Cauchy problem for non-homogeneous wave equation:

$$u_{tt} = u_{xx} - 2, \quad x \in \mathbb{R}, \quad t > 0,$$

with initial condition $u(x, 0) = \sin x$, $u_t(x, 0) = x$, $x \in \mathbb{R}$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5849 **K**

Unique Paper Code : 2352013503

Name of the Paper : Partial Differential Equations

Name of the Course : **B.Sc. (H) Mathematics – DSC-15**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and are of **15** marks each.
3. Attempt any **two** parts from each Questions. Each part is of **7.5** marks.

1. (a) Define characteristic curves of a first order partial differential equation. Find the partial differential equation arising from the two-parameter family of curves :

$$u - ax - by - ab = 0.$$

(b) Determine the integral surfaces of the equation:

$x(y^2 + u)u_x - y(x^2 + u)u_y = (x^2 - y^2)u$; with the data $x + y = 0$ and $u = 1$.

(c) Find the solution of the initial value system:

$$u_t - 2uu_x = v - x,$$
$$v_t + cv_x = 0,$$
$$u(x, 0) = x, \; v(x, 0) = x.$$

2. (a) Reduce the given equation into canonical form and find the general solution x :

$$u_x + xu_y = y.$$

(b) Using $v = \ln u$ and $v(x,y) = f(x) + g(y)$, solve the equation:

$$x^2u_x^2 + y^2u_y^2 = u^2.$$

(c) Find a complete integral of the following equation by using Charpit's method :

$$z^2 = pqxy.$$

3. (a) Derive the three-dimensional Laplace equation:

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{d^2 V}{\partial z^2}=0,$$

where V is Gravitational potential at any point.

(b) Determine the region of the following equation, and transform it to canonical form:

$$x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x.$$

(c) Derive the equation of traffic flow :

$$\frac{\partial \rho}{\partial t}+\frac{\partial q}{\partial x}=0,$$

where $\rho(x, t)$ is traffic density and $q(x, t)$ is traffic flow at any time t.

4. (a) Find the general solution of the equation:
$yu_{xx} + 3yu_{xy} + 3u_x = 0, \; y \neq 0.$

(b) Reduce the following equation into canonical form :
$u_{xx} - 2\cos x \, u_{xy} + (1 + \cos^2 x)u_{yy} + u = 0.$

(c) Find the general solution of the equation:

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-2\frac{\partial^2 z}{\partial y^2}=e^{x+y}.$$

5. (a) Determine the solution of the initial-value problem for an infinite string with initial condition:

$$u_{tt} - u_{xx} = 0, \quad x \in \mathbb{R}, \quad t > 0,$$
$$u(x,0) = \sin 3x, \; x \in \mathbb{R};$$
$$u_t(x,0) = \cos 3x, \; x \in \mathbb{R}.$$

(b) Find the solution of the initial-value problem for a semi-infinite string with fixed end:

$$u_{tt} - 16u_{xx} = 0, \quad 0 < x < \infty, \; t > 0,$$
$$u(x,0) = \sin x, \quad 0 \le x < \infty,$$
$$u_t(x,0) = x^2, \quad 0 \le x < \infty,$$
$$u(0,t) = 0, \quad 0 \le t < \infty.$$