7. (a) An urn contains n cards marked from 1 to n. Two cards are drawn at a time. Find the mathematical expectation of the product of the numbers on the cards.

(b) A random variable X has a distribution

$$f(x) = \begin{cases} \frac{1}{6}, & -3 < x < 3 \\ 0, & otherwise \end{cases}.$$

Find the distribution function and the pdf of $Y = 2X^2 - 3$. (7½×2)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5834 **K**

Unique Paper Code : 2372011102

Name of the Paper : Introduction to Probability-DSC-2

Name of the Course : **B.Sc. (H) Statistics (NEP-UGCF)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **5** questions in all.
3. Question No. **1** is compulsory.
4. Attempt **4** more questions selecting any **two** questions each from **Section A** and **Section B**.

1. Attempt any **six** :

(i) State classical and empirical definitions of probability and discuss their limitations. Explain how does the axiomatic approach overcome these limitations?

(ii) For any three events A, B and C show that

$$P(A \cap C/B) + P(A \cap C^c/B) = P(A / B)$$

(iii) For n events $A_1, A_2, \ldots, A_n$ show that

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

(iv) Three dice are thrown simultaneously. Find the probability of getting sum of the digits at least 15.

(v) A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

(vi) A man is known to speak truth 3 out of 5 times. He throws a die and reports that the number obtained is a six. Find the probability that the number obtained is actually a six.

(ii) Find the mean and variance of the above distribution. (7½×2)

6. (a) What is the expectation of the numbers of failure preceding the first success in an infinite series of trials with constant probability p of success in each trial?

(b). The length of time (in minutes) that a certain lady speaks on telephone is found to be a random phenomenon, with probability function as specified below

$$f(x) = \begin{cases} Ae^{-x/5}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

(i) Find the value of A that makes f(x) a pdf.

(ii) What is the probability that number of minutes she will talk over the phone is

(a) more than 5 minutes,

(b) less than 10 minutes

(c) between 5 and 10 minutes. (7½×2)

Section B

(Attempt any **two***)*

5. (a) The probability mass function of a random X is given by

$$P(X=i)=\begin{cases} C(\frac{1}{3^i}), & for\ i=0,1,...,6. \\ 0, & otherwise \end{cases}$$

(i) Find C.

(ii) $P(1 \le X < 4)$.

(iii) Distribution function F(x) with largest x such that $F(x) < 1/2$.

(b) The distribution of random variable X in the range (0,2) is defined by :

$$f(x)=\begin{cases} x^3, & 0<x\le 1 \\ 3(2-x)^3, & 1<x\le 2 \end{cases}$$

(i) Verify the area under the curve is unity.

(vii) A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$P(X=x)$	3k	5k	7k	9k	11k	13k

Determine the value of k? Also evaluate $P(X<4)$, $P(X \ge 5)$ and $P(3 < X \le 6)$. What is the smallest value of x for which $P(X \le x) > ½$.

(viii) In a lottery m tickets are drawn out of n tickets numbered from 1 to n. What is the expectation of the sum of the squares of numbers drawn?

(ix) A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x)=\begin{cases} 5(1-x)^4, 0<x<1 \\ 0 \quad\quad\quad , otherwise \end{cases}$$

what must the capacity of the tank be so that the probability of the supply's being exhausted in a given week is .01? (5×6)

Section A

(*Attempt any* **two**)

2. (a) For independent events A_i, i = 1, 2,..., n with respective probability of occurrence p_i. Find the probability of occurrence of atleast one of them.

   (b) Distinguish between pair-wise and mutually independent events. If A, B and C are random events in a sample space and if A, B and C are pair-wise independent and A is independent of BUC, then show that A, B and C are mutually independent. (7½×2)

3. (a) A, B and C are three urn which contain 2 white, 3 black; 3 white, 2 black; and 3 white and 3 black balls respectively. One ball is drawn from um A and put into the um B. Then a ball is drawn from um B and put into um C. Then a ball is drawn from um C. Find the chance that the ball drawn is white.

   (b) For independent events A_1, A_2,..., A_n with $P(A_i)=\frac{1}{i+1}, i=1,2,...,n.$ Find the probability that none of the event occurs, justifying each step in your calculation. (7½×2)

4. (a) State and Prove Baye's theorem and also discuss its significance.

   (b) An insurance company receives policies from three different insurers: Company 1 (40%), Company 2 (25%), and Company 3 (35%). The probability that a policyholder will file a claim in a given year is 0.50 for Company 1, 0.20 for Company 2, and 0.10 for Company 3. If a random policyholder has filed a claim, what is the probability that their policy is from Company 3? (7½×2)