SET - A **5832**

Unique Paper code : **2222011102**

Name of the Paper : **Mechanics**

Name of the Course : **B.Sc. Hons. (Physics)_NEP: UGCF**

Semester : **I**

Duration: **3 Hours** Maximum Marks: **90**

Instruction for candidates

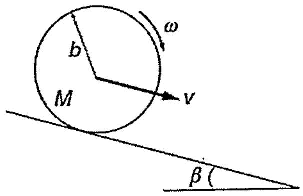
1. Write your Roll no. on the top immediately on receipt of this question paper.
2. **All** Questions carry equal marks.
3. Question **No. 1** is compulsory.
4. Answer **any four** of the remaining five questions.
5. Use of non-programmable scientific calculators are allowed.

1. Attempt **all** parts of this question. $(3 \times 6 = 18)$

(a) Show that in centre of mass frame of reference, the magnitude of the velocities of the particles remains unaltered in elastic collision.

(b) Consider the first stage of a rocket used for the moon landing program. The initial mass is $2.8 \times 10^6\ kg$, and the mass of the first-stage fuel is $2.1 \times 10^6\ kg$. Assume a mean thrust of $37 \times 10^6\ N$. The exhaust velocity is $2600\ m/s$. Calculate the final speed of the first stage at burnout.

(c) Consider a uniform drum of radius b, mass M, weight $W = Mg$, and moment of inertia $I_o = Mb^2/2$ on a plane of angle β. If the drum starts from rest and rolls without slipping, find the speed V of its center of mass after it has descended a height h.



(d) Calculate the value of the centrifugal force on a mass of $20\ g$ placed at a distance of $10\ cm$ from the axis of a rotating frame of reference, if the angular speed of rotation of the frame be $10\ rad/s$.

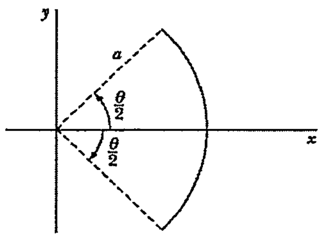
(e) Show that the angular momentum of a particle moving under a central force is conserved.

(f) A proton of rest mass $1.67 \times 10^{-24}\ g$ is moving with velocity $0.9c$. Find its relativistic mass and momentum.

2. (a) A rocket ascends from rest in a uniform gravitational field by ejecting exhaust gases with a constant speed u relative to the rocket. Assuming that the rate at which mass is expelled is given by $\frac{dm}{dt} = -\gamma m$, where m is the instantaneous mass of the rocket and γ is a constant, show that the velocity of the rocket as a function of time is given by

$$v(t) = (\gamma u - g)t.$$

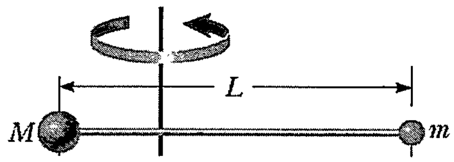
(b) Find the center of mass of a uniform wire that subtends an arc θ of the radius of the circular arc is a.



(c) A particle of mass m collides elastically with a particle of mass $3m$ which is at rest. In C –frame, the particle of mass m moves with speed $3\,m/s$ at an angle of 60^o after the collision. Find the final velocity of mass m in the L –frame and the fractional loss of kinetic energy of the mass m.

(7, 6, 5)

3. (a) Two masses M and m are connected by a rigid rod of length L having negligible mass. For an axis perpendicular to the rod, show that the system has the minimum moment of inertia when the axis passes through the center of mass which is given by $I_{min} = \left(\frac{mM}{m+M}\right)L^2$.

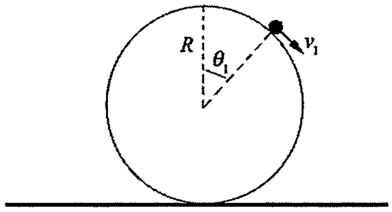


(b) A particle of mass m is subjected to a one-dimensional force $\vec{F} = (-kx + bx^3)\hat{x}$, where k and b are positive constants.

(i) Find the equilibrium points and classify these points as stable or unstable equilibrium.

(ii) Find the turning points for energy $E = 3k^2/16b$.

(c) A small object of mass m rests on top of a sphere of radius R. The object is released from the top of the sphere with a negligible speed and it slowly starts to slide. Determine the angle θ_1 with respect to the vertical at which the object will lose contact with the surface of the sphere.



(7, 6, 5)

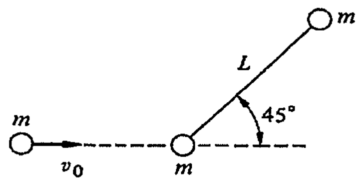
4. (a) What are central forces? Introducing the idea of reduced mass, show that a two-body problem under central forces can be reduced to a one-body problem.

(b) A particle of mass m moves under a central force whose potential energy function is $U(r) = kr^4, k > 0$.

(i) For what energy and angular momentum will the orbit be a circle of radius 'a' about the origin?

(ii) If the particle is slightly disturbed from this circular motion, what will be the period of small radial oscillations about $r = a$?

(c) A rigid massless rod of length L joins two particles each of mass m. The rod lies on a frictionless table, and is struck by a particle of mass m and v_o, moving as shown in figure below. After collision, the projectile moves straight back. Prove that the angular speed of the rod about its centre of mass after the collision is given by $\omega = \frac{4\sqrt{2}}{7}\frac{v_o}{L}$.



(7, 6, 5)

5. (a) Considering the rotating Earth to be a sphere of radius R having angular speed ω, show that for a body at rest at a point on the Earth's surface, the effective value of g at latitude λ is, approximately, given by

$$g_{eff} = g_o - \omega^2 R \cos^2 \lambda$$

where g_o is the true acceleration due to gravity.

(b) Show that $\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ is a Lorentz invariant. Explain space-like, time-like and light-like events.

(c) Calculate the magnitude and direction of the Coriolis acceleration of a rocket moving upward at 2 km/s at 60^o South latitude.

(7, 6, 5)

6. (a) Deduce Einstein's mass-energy equivalence relation $E = mc^2$. Give any two experimental evidences showing its validity.

(b) What is length contraction. How fast should a rocket ship move relative to an observer so that its observed length appears to be 99% of its proper length?

(c) The proper lifetime of an elementary particle is $1.0 \times 10^{-8}\, s$. Find the lifetime as measured in the laboratory frame, where its velocity is observed to be $0.8c$.

(7, 6, 5)