[This question paper contains 2 printed pages.]

Your Roll No...............

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Sr. No. of Question Paper	**:**	**5815**
Unique Paper Code	:	2372012302
Name of the Paper	:	Advanced Probability Distributions, DSC – 8
Name of the Course	:	**B.Sc. (Hons.) Statistics (Under NEP - UGCF)**
Semester	:	III

Duration : 3 Hours — Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **two** questions from each section.
3. **All** questions carry equal marks.

Section: A

1. (a) Derive the cumulant generating function of negative binomial distribution and hence obtain first four cumulants, β_1 and β_2.

   (b) Let X is a negative binomial variate with p.m.f.,

$$f(x) = \begin{cases} \binom{k+x-1}{x} q^x p^k, & x = 0,1,2 \ldots \\ 0, & \text{otherwise.} \end{cases}$$

   Show that the moment recurrence formula is:

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\,\mu_{r-1}\right).$$

   (7,8)

2. (a) In an infinite sequence of independent Bernoulli trails with constant probability 'p' of success for each trail, find the mean and variance of the number of trails required to get the first success.

   (b) Explain how and under what conditions the hypergeometric distribution can be approximated by the binomial distribution. **(8,7)**

3. (a) Define multinomial distribution. Obtain the formula for the correlation coefficient between two random variables following a multinomial distribution.

   (b) Two discrete random variables X and Y have the joint probability distribution:

$$p(x,y) = \frac{9!}{x!\,y!\,(9-x-y)!}\left(\frac{1}{3}\right)^9.$$

   For $x, y = 0, 1, 2, \ldots, 9$ and $0 \le x+y \le 9$. Show that

   (i) Show that the marginal distribution of X is Binomial with parameters $n = 9$ and $p = 1/3$.

   (ii) Show that the conditional distribution of Y given X=3, is also binomial with parameters $n = 6$ and $p = 1/2$. **(7,8)**

Section-B

4. (a) If X has a uniform distribution in [0,1]. Find the pdf of $-2\log X$, and also identify the distribution.

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(b) Suppose that probability function of a random variable X is given by:

$$f(x) = \begin{cases} \frac{x^k . e^{-x}}{k!}; for\ x > 0\ and\ k > 0 \\ 0; \quad otherwise \end{cases}$$

(i) Find the moment generating function of this distribution.

(ii) Determine the mean and variance of this distribution using moment generating function. **(7,8)**

5. (a) Suppose that X is a random variable for which $E(X) = \mu$ and $Var(X) = \sigma^2$. Further suppose that Y is uniformly distributed over the interval (α, β). Determine α and β, so that $E(X) = E(Y)$ and $Var(X) = Var(Y)$.

(b) Show that the mean value of positive square root of a $\gamma(\mu)$ variate is $\gamma\left(\mu + \frac{1}{2}\right)/\Gamma(\mu)$. Hence prove that mean deviation of a $N(\mu, \sigma^2)$ variate from its mean is $\sqrt{2/\pi}$. **(7,8)**

6. (a) If X & Y are independent Gamma variates with parameter μ & v respectively. Find the distribution of $U = X + Y$, and $Z = \frac{X}{X+Y}$. Also show that they are independently distributed.

(b) Define beta distribution of first kind. Compute r^{th} moment and harmonic mean for beta distribution of first kind. **(8,7)**

Section-C

7. (a) Find the characteristic function of standard Laplace distribution and hence find its mean and standard deviation

(b) Let X has a standard Cauchy distribution. Find the p.d.f for X^2 and identify the distribution. **(7,8)**

8. (a) A sample of n values is drawn from a population with p.d.f.

$$f(x) = \theta e^{-\theta x}, (x > 0, \theta > 0)$$

If $\bar{X}$ is the mean of the sample, show that $n\theta\bar{X}$ is a $\gamma(n)$ variate and prove that

$$E(\bar{X}) = \frac{1}{\theta} \quad \text{and} \quad S.E.(\bar{X}) = \frac{1}{\theta\sqrt{n}}.$$

(b) Obtain the moment generating function of Logistic distribution and hence find its mean and variance. **(8,7)**

9. (a) If $X_1, X_2, X_3, \dots, X_n$ be i.i.d. random sample from a population with continuous density. Show that, $Y_1 = \min(X_1, X_2, X_3, \dots, X_n)$ is an exponential distribution with parameter $n\theta$ if and only if each X_i is exponential with parameter θ

(b) If $X \sim N(\mu, \sigma^2)$, then find the distribution of $U = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$. **(8,7)**