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- (c) List all the members of dihedral group D_8 . Find two objects, images or shapes that have same number and symmetries of as D_8 .

(2000)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 13446 **K**

Unique Paper Code : 2354000012

Name of the Paper : APPLIED ALGEBRA

Name of the Course : **COMMON PROG GROUP (GE)**

Semester : VII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Consider the production model $x = Cx + d$ for an

economy with two sectors, where $C = \begin{bmatrix} .2 & .4 \\ .5 & .3 \end{bmatrix}$ and

P.T.O.

$d = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$. Use an inverse matrix to determine the production level necessary to satisfy the final demand.

(b) Find the 3×3 matrix that corresponds to the composite transformation of a scaling by .2, a rotation of 45° about the origin, and finally a translation that adds $(1, -3)$ to each point of a figure.

(c) Verify that $(-1)^k$, 2^k and $(-2)^k$ are linearly independent signals.

2. (a) Verify that the signal $y_k = -2$ satisfies the difference equation $y_{k+2} - y_{k+1} - 6y_k = 12$ for all k . Then find a description of all solutions of this equation.

(b) Find a least-squares solution of the inconsistent

$$\text{system } Ax = b \text{ for } A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}.$$

*	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	0	6	7	8	9	5
2	2	3	4	0	1	7	8	9	5	6
3	3	4	0	1	2	8	9	5	6	7
4	4	0	1	2	3	9	5	6	7	8
5	5	9	8	7	6	0	4	3	2	1
6	6	5	9	8	7	1	0	4	3	2
7	7	6	5	9	8	2	1	0	4	3
8	8	7	6	5	9	3	2	1	0	4
9	9	8	7	6	5	4	3	2	1	0

Does this scheme catch all single digit errors? Prove it.

(b) Define IBM check digit scheme? Use IBM check digit scheme to determine which is valid or invalid credit card no:— 000013017698 130922781724.

Below are two transpositions of adjacent digit errors

Correct number : 190322 211490

Incorrect number : 910322 214190

Which of these will be caught and what is the reason thereof?

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- (c) Two check digit schemes are proposed for a 10-digit number $a_1 a_2 a_3 a_4 a_5 \dots a_{10}$

1. Scheme A: $\sum_{i=1}^{10} a_i \equiv 0 \pmod{10}$

2. Scheme B: $\sum_{i=1}^{10} i a_i \equiv 0 \pmod{11}$

Which scheme is better at detecting single digit errors and transposition of adjacent digits errors? Provide a mathematical justification for your conclusion.

6. (a) Define Verhoeff check digit scheme. If a correct number 386018429278 incurred a single digit error and was transmitted as 386016429278. Would it be caught by Verhoeff scheme?

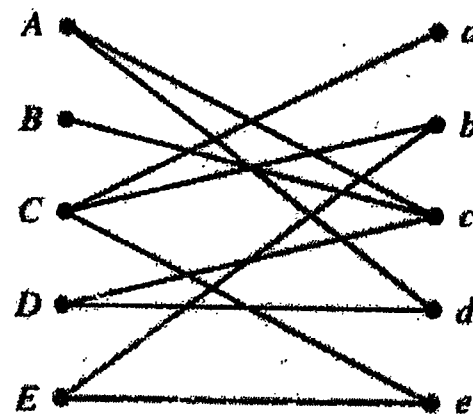
Use $\sigma = (0)(1,4)(2,3)(5,6,7,8,9)$ and use following table :

- (c) Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points (2,1), (5,2), (7,3) and (8,3).

3. (a) Define Latin squares and orthogonal Latin squares. Give examples of Latin square and orthogonal Latin squares of order 3.

- (b) Construct and verify a Latin square for the group $\mathbb{Z}_7$ with addition modulo 7.

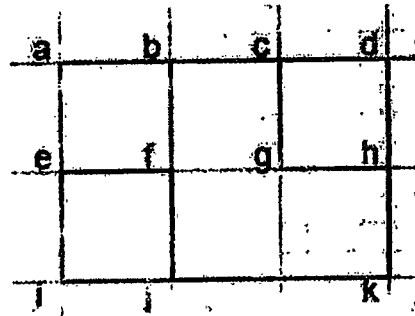
- (c) For the graph below with five persons A, B, C, D, E and five jobs a, b, c, d, e, find a matching or explain there does not exist a matching.



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4. (a) For the graph below with vertices a, b, c, d, e, f, g, h, i, j, k, find all the pair of edges that will make the graph disconnected.



- (b) In a tournament, there are twelve teams and each team plays with two other teams. Draw the graph of the schedule of the tournament?
- (c) Suppose there are given three containers of water, of sizes 10 liters, 7 liters, and 4 liters. Initially the 10-liter container is full. Two other containers are empty. One can pour water from one container into another, pouring until the receiving pitcher is full or the pouring pitcher is empty. Is there a way to pour 2 liters of water in each of the 7-liter and 4-liter containers? If yes, find a minimal sequence of pourings to do so.

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5. (a) Use Sieve of Eratosthenes to find all prime numbers between 50 and 80.

Calculate $(17^8 \times 25^7) \pmod{7}$.

UPC of a product is 0-5360Y-10054-0. 6th digit is unknown in this no. Find the missing digit Y.

- (b) Using airline ticket check digit scheme, assign check digit to the following identification number :

0-004-2871911233

Using ISBN check digit scheme, determine which of the following is a valid or invalid ISBN number:

0-471-51001-7

3-824-27519-X

Define ISBN check digit scheme? What are the two drawbacks of this ISBN scheme?

P.T.O.