[This question paper contains 4 printed pages.]

Your Roll No................

Sr. No. of Question Paper : 9100 **K**

Unique Paper Code : 2373010015 (NEP-UGCF)

Name of the Paper : Financial Statistics (DSE)

Name of the Course : **BSc. (Hons.)**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **TWO** sections in this paper.
3. **All** sections are compulsory.
4. Use of non-programmable scientific calculator is allowed.

Section I (Attempt Any FIVE parts)

1. (a). What is internal rate of return? Explain with the help of example.

   (b). An 8% bond with 18 years to maturity has a yield of 9%. What is the price of this bond if the coupon payments are made (i) annually (ii) half yearly and face value is $1,000?

   (c). Define 'intrinsic value' of a call and put option at time t. Under what conditions is an option said to be 'in-the-money', 'at-the-money'; and 'out-of-money'?

   (d). One of the most popular types of spread is a bull spread. A bull call-price spread can be made by buying a call option with a certain strike price and selling a call option on the same stock with higher exercise price. Both call options have same maturity date. Consider a European call with a delivery price of K_1 and second European call with delivery price of K_2. Draw the payoff for this strategy.

   (e). Consider a butterfly strategy: a long call option with an exercise price of 100 USD, a second long call option with an exercise price of 120 USD and two short calls with an exercise price of 110 USD. Give the payoff table for different stock values. When will this strategy be preferred?

(f). Consider a long forward contract on a 5-year bond currently trading at a price of ₹ 900. The delivery price is ₹910, the time to maturity of the forward contract is 1 year. The coupon payment of the bond of ₹ 60 occurs after 6 and 12 months. The continuously compounded annual interest rate for 6 and 12 months are 9% and 10% respectively. Obtain the value of forward contract.

(g). Derive $\int_0^t W_s^2 dW_s$, where W_t is a Brownian motion.

(5 × 3)

Section II (Attempt Any FIVE questions)

2. (a) The projected cashflows in (₹) for a multinational company from a 4-year project are as follows:

Initial cost	4,68,000
Year	Cashflow
Ist Year	1,35,000
IInd Year	2,40,000
IIIrd Year	1,85,000
IVth Year	1,35,000

Conclude whether the company project manager should accept or reject the project on using a discount rate of (i) 8% and (ii) 20% for this project.

(b) Consider a long forward contract to buy a stock which has a price of S_t at time t. Let K be the delivery price and T be the maturity date. Further let $V(t, S_t)$ denote the value of the long forward contract at time t as a function of the current price $S_t = s$. The time to maturity is $\tau = T - t$. Assume that the interest rate is constant during the time to maturity. The stock does not pay any dividend and does not involve any costs during the time to maturity T.

Then show that

$$V(t, S_t) = V_{K,T}(t, S_t) = S_t - Ke^{-r\tau}$$

Hence obtain the forward price.

(7,8)

3. (a) The correlation (ρ) between two assets 'A' and 'B' is 0.1. The expected returns and standard deviations of returns are given in the following table:

Asset	Return (%)	Standard deviation (%)
A	10	15
B	18	30

(i) Find the proportions α and (1-α) of 'A' and 'B' respectively that defines a portfolio consisting of them such that it has minimum variance.

(ii) Find the expected return and the standard deviation of this portfolio.

(b) Construct the payoff table along with the payoff graph for the following two alternative strategies:

Strategy 1: consists of buying a European call option with a strike price \$110 and selling a call option on the same stock with higher strike price of \$120. Both call options have same expiration date.

Strategy 2: consisting of a long call with delivery price \$40 and a long put with delivery price \$40; and same expiration date.

(7,8)

4. (a) Consider a European call option on a stock with current spot price S_0 = 20, dividend D = 2 USD, exercise price K = 18 and time to maturity of 6 months. The annual risk-free rate is r = 10%. What is the upper and lower bound (limit) of the price of the call options?

(b) Obtain the expression for the Greek 'Theta' of a European put option under the Black–Scholes model.

(7,8)

5. (a) Define a Wiener process. Let $\{W_t; t \geq 0\}$ be a standard Wiener process. Then show that the process defined as $X_t = \begin{cases} W_t; & t \leq T \\ 2W_T - W_t; & t > T \end{cases}$ is also a Wiener process.

(7,8)

6. (a) State and prove Itô's Lemma for a differentiable function $Y_t = g(X_t)$, where $\{X_t, t \geq 0\}$ is an Itô process governed by the stochastic differential equation

$$dX_t = \mu(X_t, t)\, dt + \sigma(X_t, t)\, dW_t,$$

with $\{W_t; t \geq 0\}$ denoting a standard Wiener process.

(b) (i) Assuming that the stock price process is geometric Brownian motion with $\frac{dS_t}{S_t} = \mu dt + \sigma\, dW_t$, where $\{W_t; t \geq 0\}$ is Wiener process, obtain dynamics of the log-price process $Y_t = G(S_t) = log(S_t)$.

(ii) Further compute the following: Suppose ABC bank's current stock price is $S_0 = ₹950$; the expected return and volatility are 10% and 25% per annum respectively. Then, identify the probability distribution for $Y_t = G(S_t)$ for $t \in [0, T]$.

(7,8)

7. (a) Let $F = F(S_t, t)$ denote the value of a derivative written on a stock whose stock price S_t and time t follows an Itô process given by

$$dS_t = \mu\, S_t\, dt + \sigma\, S_t\, dW_t,$$

where μ is the drift rate and σ is the volatility.

Show that the following relationship holds in the usual notations of option pricing: $rF = \Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma$.

(b) Calculate the delta of an at-the-money nine-month European call option. Given that the underlying stock is non-dividend paying, the risk-free interest rate is 5% per annum and the volatility of the stock price is 29% per annum.

Table of Φ(z) or CDF of N[0,1] values

z =	0.10	0.20	0.25	0.28	0.30	0.35	0.40
Φ(z) =	0.5398	0.5792	0.5987	0.6102	0.6179	0.6368	0.6554

(7,8)