[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	**:**	**13537**	**K**
Unique Paper Code	:	2374000019	
Name of the Paper	:	GE: Elements of Stochastic Process	
Name of the Course	:	**B.Sc. (Hons.) Statistics**	
Semester	:	VII (NEP-UGCF)	

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions. Question No. **1** is compulsory.
3. Attempt **four** questions from the remaining questions.
4. **All** questions carry equal marks.
5. Use of a non-programmable scientific calculator is allowed.

1. Attempt any ***Three*** parts.

(a) If $G_X(s)$ is the p.g.f. of X, what are the expectation and variance of X? Show the derivation.

(b) Let $\{X_n, n \geq 1\}$ be a sequence of uncorrelated random variables with mean 0 and variance 1. Is $\{X_n, n \geq 1\}$ a covariance stationary process? Explain.

(c) Let us suppose that the customers arrive (singly) at a bank in accordance with a Poisson process at a rate of 3 per minute, then in a duration of 2 minutes. What is the probability that the number of customers arriving is 4?. Give reasons for your answer.

(d) Let $Y_n = a_1X_{n-1} + a_0X_n$, $(n = 1, 2, ...)$, where a_0, a_1 are constants and X_n , $n = 0, 1, 2, ...$ are i.i.d. random variables with mean 0 and variance σ^2. Is $\{Y_n, n \geq 1\}$ covariance stationary? Explain.

2. (a) Let $S_N = X_1 + X_2 + \cdots + X_N$ where X_i's are i.i.d. random variables with p.g.f. P(s) and N is a random variable independent of X_i's with p.g.f. G(s). What is the p.g.f. of S_N. Give a detailed proof.

(b) Using the result in part (a) obtain the p.g.f. of a Compound Poisson distribution. Clearly indicate the concerned notations.

3. (a) What is a Markov chain? Explain with examples. What do you mean by the transition probability matrix of a Markov chain? What are its properties?

(b) Consider a Markov chain $\{X_n, n \geq 0\}$ such that X_n assumes values -1 and 1 with conditional probabilities:

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0<a, b<1$$

and initial distribution $P\{X_0 = 1\} = p_1 = 1 - P\{X_0 = -1\}$.

If $P\{X_n = 1\} = p_n$, find:

i) $E\{X_n\}$ **ii)** $Var\{X_n\}$ **iii)** first-order $Cov\{X_{n-1}, X_n\}$
iv) $Cov\{X_{n-1}, X_n\}$ when a=b.

4. (a) Obtain the p.g.f. of the following distributions. Hence obtain their expectation and variance.
(i) Binomial (n,p) distribution.
(ii) Geometric (p) distribution.

(b) Explain clearly what you mean by a reducible Markov chain.

5. (a) Define a Poisson process. State the postulates that characterize a Poisson process and derive the probability law of a Poisson process $\{N(t), t \geq 0\}$. Also show that

$$P\{N(t) = n\} = e^{-\lambda t}\frac{(\lambda t)^n}{n!}, \quad n = 0, 1, 2, ...$$

Also, prove that the mean and variance of $N(t)$ are both equal to λt.

(b) Suppose the number of customers arriving at a shop follows a Poisson process with rate 5 per hour. Each customer independently buys with probability 0.6.

(i) Find the distribution and mean of the number of buyers in 2 hours.
(ii) Find the probability that exactly 3 customers buy.

6. (a) If $\{N(t)\}$ is a Poisson process, then show that the autocorrelation coefficient between N(t) and $N(t+s)$ is $\left(\frac{t}{t+s}\right)^{1/2}$.

(b) Let $\{X_n, n \geq 0\}$ be a Markov chain having state space S={0, 1, 2, 3}and transition matrix as:

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.3 & 0.2 & 0.5 & 0 \\ 0 & 0.2 & 0.3 & 0.5 \end{bmatrix}.$$

i) Is the chain irreducible?
ii) Find the nature of all states.