SECTION II

5. Consider the simple linear regression model $Y = \beta_0 + \beta_1 X + \in$ with usual assumptions.

   (i) Show that the sum of the residuals weighted by the corresponding value of the regressor variable is always equal to zero.

   (ii) Find a $100(1-\alpha)\%$ confidence interval for β_1.

   (iii) Show that $\mathrm{Cov}\left(\hat{\beta}_0, \hat{\beta}_1\right) = -\bar{x}\dfrac{\sigma^2}{S_{xx}}$. (5,5,5)

6. (a) Discuss the problem of testing for lack of fit in simple linear regression model.

   (b) For the multiple regression model, derive the test statistics for testing the significance of individual regression coefficients. (6,9)

7. Describe the technique of analysis of variance. Derive the analysis of variance of two way classified with one observation per cell under fixed effects model. (15)

8. Write short notes on :

   (i) No intercept regression model

   (ii) Coefficients of determination

   (iii) Orthogonal Polynomials (5,5,5)

(1000)

[This question paper contains 2 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 9737 **K**

Unique Paper Code : 2373570006

Name of the Paper : DSE : Introduction to Statistical Linear Models

Name of the Course : **B.A. / B.Sc. (P) under NEP-UGCF**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **Six** questions in all, selecting **Three** from each Section.

3. Attempt all parts of question in continuation.

4. Use of non-programmable scientific calculator is allowed.

P.T.O.

SECTION I

1. (a) Describe the general linear model and discuss briefly the different studies derived from it.

   (b) Consider the models $Y_1 = \beta_1 + \beta_2 + \epsilon_1$, $Y_2 = \beta_1 + \beta_3 + \epsilon_2$, and $Y_3 = \beta_1 + \beta_2 + \epsilon_3$ with usual assumptions. Prove that $\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ is estimable if and only if $\lambda_1 = \lambda_2 + \lambda_3$. (8,7)

2. (a) For a given model $Y_{n\times1} = X_{n\times p}\,\beta_{p\times1} + \epsilon_{n\times1}$ with $E(\epsilon) = 0$, $V(\epsilon) = \sigma^2 I$ and $R(X) = p < n$. Obtain an unbiased estimator of σ^2.

   (b) Consider the models $Y_1 = \beta_1 + \beta_2 + \beta_3 + \epsilon_1$, $Y_2 = \beta_1 + \beta_3 + \epsilon_2$ and $Y_3 = \beta_2 + \beta_3 + \epsilon_3$, $Y_4 = \beta_2 + \epsilon_4$ with usual assumptions.

   (i) Write the above general linear model in matrix notation.

   (ii) Obtain BLUE of the unknown parameters along with its variance-covariance matrix. (6,9)

3. Let $Y = (Y_1, Y_2, \ldots, Y_n)'$ be a vector of n independent standard normal variates. Let $Q_1 = Y'A_1Y$ and $Q_2 = Y'A_2Y$ be distributed as χ^2 with n_1 and n_2 d.f. respectively. Prove that the necessary and sufficient condition for Q_1 and Q_2 to be independently distributed is $A_1A_2 = 0$. (15)

4. (a) Let $Y \sim N(\mu, \Sigma)$, where $\mu' = (2\ -1\ 3)$ and $\Sigma = 1$.

   Let $A = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$. Find expected value of $Y'AY$. Does $Y'AY$ have a chi-square distribution?

   (b) Four objects A, B, C, D are involved in a weighing experiment. Put together they weighed Y_1 grams; when A and C are put in the left pan of the balance and B and D are put in the right pan, a weight of Y_2 grams was necessary in the right pan for the balance. With A and B in the left pan and C and D in the right pan, Y_3 grams were needed in the right pan. Finally A and D in the left pan and B and C in the right pan, Y_4 grams were needed in the right pan. If the observations Y_1, Y_2, Y_3, Y_4 are all subject to uncorrelated errors with common variance σ^2, obtain the BLUE of individual weights and the total weight of the four objects, and variance of the estimate of total weight of four objects. (8,7)