[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 9733 **K**

Unique Paper Code : 2373010015 (NEP-UGCF)

Name of the Paper : Financial Statistics (DSE)

Name of the Course : **BSc. (Hons.)**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **TWO** sections in this paper.
3. **All** sections are compulsory.
4. Use of non-programmable scientific calculator is allowed.

Section I (Attempt Any FIVE parts)

1. (a). Define IRR. Hence, which of the two cash flow streams would you prefer and why: $(-2,1,1,1)$ and $(-2,0.5,0.5,2.5)$.

(b). Consider the following binary one-period model: a stock with price $S_0 = 275$, a European call option on a stock with strike price 275, a zero bond with price $1/(1+r)$ with interest rate 5%. The stock can increase to 305 or decrease to 255. Calculate the price of the call C_0.

(c). Define a duplicating portfolio. Consider two portfolios A and B which have the same value at a certain time T. Show that both the portfolios must have the same value at any time prior to T as well.

(d). Define the strategy called 'straddle' and draw the corresponding payoff table.

P.T.O.

(e). If a bank offers sweep-account interest at a nominal rate of 6.5% p.a. compounded continuously, what is the effective interest rate per year?

(f). Obtain the expression for 'delta' of an European call options.

(g). Let $\{W_t; t \geq 0\}$ be a Wiener process. Is $X_t = W_t^2 - t$ a martingale with respect to the natural filtration F_t associated with W_t?

(5 × 3)

Section II (Attempt Any FIVE questions)

2. (a) What are perpetual bonds? A perpetuity entitles its holder to be paid the constant amount c at the end of each of an infinite sequence of years. If the interest rate is r, compounded yearly, then what is the present value of such a cash flow sequence?

(b) An investor has a capital of ₹ 10,500 at his disposal to buy stocks whose current price is ₹100. Further put options on the same stock with a delivery price of K= ₹100 and a time to maturity of 1 year are quoted at a market price of ₹5 per contract. Consider two alternative investment plans:

Portfolio A : Buying 105 stocks

Portfolio B : Buying 100 stocks for 10,000 and buying 100 put options for 500.

Discuss the effect of portfolio insurance on portfolio value and return.

(6,9)

3. (a) Show that the price of an American or European put option is the convex function of the delivery price.

(b) If interest rates are constant during contract period, then show that the forward and the future prices are equal.

(6,9)

4. (a) (i) Two stocks are available. The corresponding expected rates of return are $\bar{r}_1$and $\bar{r}_2$. The respective variances and covariance are, σ_1^2, $\sigma_2^2$and σ_{12}. Construct a portfolio consisting of the two stocks so that it has the minimum variance in the class of all portfolios consisting of these two assets. What is the mean rate of return of this portfolio?

(ii) Consider the returns are 10% and 15%; variances are 20% and 30% respectively. Determine σ_{12} if the mean return of this portfolio is 12%.

(b) Consider a long forward contract to buy an asset whose price at time t is S_t. Let K denote the delivery price and T the maturity date. Assume constant risk-free interest rate r over the period to maturity. Let the underlying asset pay discrete dividends or incur costs during the period to maturity, whose total present value at time t equals D_t.

(i) Derive an expression for the value of the long forward contract at time t, denoted by $V(S_t, \tau)$, where $\tau = T - t$.

(ii) Hence, obtain the formula for the forward price F_t.

(7,8)

5. (a) Define a Wiener process. Let $\{W_t; t \geq 0\}$ be a standard Wiener process. Show that the process defined as $X_t = W_{T+t} - W_t; T > 0$ is also a Wiener process.

(b) Let $\{W_t; t \geq 0\}$ be a standard Wiener process. Derive the value of $E(W_s^4)$.

(7,8)

6. (a) State and prove Itô's Lemma for a differentiable function $Y_t = g(X_t, t)$, where $\{X_t, t \geq 0\}$ is an Itô process governed by the stochastic differential equation

$$dX_t = \mu(X_t, t)\,dt + \sigma(X_t, t)\,dW_t,$$

with $\{W_t; t \geq 0\}$ denoting a standard Wiener process.

(b) For a put option whose price $P = P(S_t, t)$ is a function of the stock price S_t and time t, assume that the stock price follows an Itô process given by

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t,$$

where μ is the drift rate and σ is the volatility.

Show that in the usual notations of option pricing, $rP = \Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma$, where $\Delta = \frac{\partial P}{\partial S}$, $\Gamma = \frac{\partial^2 P}{\partial S^2}$ and, $\Theta = \frac{\partial P}{\partial t}$.

(6,9)

7. (a) Assume a put option with exercise price $K = 8$ at $T = 2$, which is to be priced at $t = 0$. The current stock price is 10 which is expected to increase or decrease by 20% in the first period. For the second period, the stock price is expected to increase or decrease by 10%. Assuming the risk-free rate of interest to be 0, find the value of the put at $t = 0$.

(b) Let the current price of a stock is 50. If the risk-free rate is 10% and volatility associated with the stock price movement is 25%, find the price of a call in 3 months if the exercise price is 49. Use put call parity to determine the price of a put on this stock.

Table of Φ(z) or CDF of N[0,1] values

z =	-0.12	0.02	0.12	0.22	0.32	0.42	0.56
Φ(z) =	0.4522	0.5079	0.5477	0.5870	0.6255	0.6627	0.7122

(7,8)