[This question paper contains 4 printed pages.]

Your Roll No..............

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Sr. No. of Question Paper : 533

Unique Paper Code : 2372574701 (NEP-UGCF)

Name of the Paper : DSC: Stochastic Process and Queueing Theory

Name of the Course : **B.A. / B.Sc. (P)**

Semester : VII

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Q. No. **1** is compulsory.
4. Attempt **five** more questions from the Q. No. **2** to **7**.
5. Use of a non-programmable scientific calculator is allowed.

SECTION I

1. Attempt any **five** parts.

(a) For a (M|M|1) with FCFS service discipline and infinite system capacity queuing system, show (in usual notations)

$$P(\text{queue length} \geq n) = \rho^n$$

(b) Let X be a non-negative integral valued random variable with $P(X = k) = p_k$; $k = 0,1,2...$ and probability generating function P(s). Find the generating function of $P(X \geq k)$.

(c) Let the probability of transition from a dry day to a wet day be 0.55 and that of transition from a wet day to a dry day be 0.6. The initial probability distribution of a day being a dry day or a wet day is (0.7 0.3). Find the probability of the second day being a wet day.

(d) In the classical ruin problem find the expected gain for a player and interpret the result.

P.T.O.

(e) Show that the initial distribution together with transition matrix completely defines a Markov chain.

(f) Let X be a positive discrete random variable such that $P[X = k] = p_k$, $P[X > k] = q_k = \sum_{r=k+1}^{\infty} p_r$, $k \geq 0$ and $P(s) = \sum p_k s^k$ and $Q(s) = \sum q_k s^k$.

Obtain a relation between P(s) and Q(s). Show that $E[X] = Q(1) = \sum q_k$.

(g) Let $Y_n = a_0 X_n + a_1 X_{n-1}$ $(n = 1,2, ...)$ where a_0, a_1 are constants and X_n, $n = 0,1,2...$ are i.i.d. random variables with mean 0 and variance σ^2.

Is $\{Y_n, n \geq 1\}$ covariance stationary? (3×5)

Section II

2. (a) Let X be a random variable (r.v) denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Obtain the p.g.f. of the r.v. X.

(b) Let $S_N = X_1 + X_2 + ... + X_N$, where X_i's are i.i.d. random variables and N is a random variable independent of X_i's. Find expectation and variance of S_N. (7,8)

3. (a) Consider the process $X(t) = \sum_{r=1}^{k} (A_r \cos \theta_r t + B_t \sin \theta_r t)$ where A_r, B_r are uncorrelated random variables with mean 0 and variance σ^2 and θ_r are constants. Is the process $\{X(t), t \in T\}$ covariance stationary?

(b) A doctor can either go by a car or take a cab to go to his clinic every day. He never goes two days in a row in car , but if he goes by car one day , then the next day he is equally likely to travel by cab as he is to go by car again .On the first day of the week he throws a dice and goes by car to his clinic iff "3 " appears. Find the probability that

(i) he goes by car to his clinic on the second day of the week.

(ii) he takes a cab in the long run. (7,8)

4. (a) Consider the Markov Chain with three states, $S = \{0, 1, 2\}$ having the following transition matrix :

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

(i) Is the chain irreducible?

(ii) Find the nature of the states 0, 1 and 2.

(b) Consider a sequence $\{X_n, n\geq 2\}$ of independent coin tossing trials with probability p for head(H) in a trial. Denote the states of X_n by states 1,2,3,4 according as the trial numbers (n-1) and n results in HH,HT,TH,TT respectively. Show that $\{X_n, n\geq 2\}$ is a Markov chain and construct the transition probability matrix. (7,8)

5. (a) Find expected duration of the game in the classical ruin problem. Also obtain the expected duration of the game when a gambler plays against an infinitely rich adversary.

(b) Let $\{X_n, n \geq 0\}$ be a Markov chain with $S = \{0, 1, 2\}$, transition probability matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix} \end{array}$$

and the initial distribution $P[X_0 = i] = \frac{1}{3}$, $i = 0, 1, 2$. Find the probabilities

(i) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$.

(ii) $P(X_1 = 1)$.

(iii) $P(X_2 = 1, X_0 = 0)$ (7.8)

6. (a) State the postulates for Poisson process. Show that under these postulates N(t), the number of occurrences of an event E up to the epoch t follows Poisson distribution with mean λt.

   (b) Let N(t) be a Poisson process then show that;

   (i) $\Pr\{N(s) = k \mid N(t) = n)\} = {}^{n}C_{k}(s/t)^{k}(1-(s/t))^{n-k}$, $s < t$

   (ii) $\dfrac{N(t)}{t}$ provides a consistent estimate for mean rate λ of the Poisson process $\{N(t), t \geq 0\}$. (7,8)

7. (a) In the case of (M/M/1); (∞/FIFO) queuing model, obtain expressions for

   (i) Expected number of customers in the queue

   (ii) The probability that the number of customers is $\geq k$.

   (iii) Expected number of customers in the system

   (b) A petrol station has a single pump and space for not more than 3 cars (2 waiting and 1 being served). A car arriving when the space is filled to capacity goes elsewhere for filling-up of petrol. Cars arrive according to a Poisson distribution at a mean rate of one every 8 minutes.

   Their service time has an exponential distribution with a mean of 4 minutes.

   (i) Find the effective arrival rate.

   (ii) What is the probability that an arriving car will get service immediately upon arrival?

   (iii) Find the expected number of cars in the system. (7.8)