[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper	:	**6362**
Unique Paper Code	:	2352571101
Name of the Paper	:	DSC – Topics in Calculus
Name of the Course	:	**Bachelor of Arts / Bachelor of Science (Programme) with Mathematics as non- major / minor**
Semester	:	I

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Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1.

a) Test the continuity at $x = 0$ of the following function:

$$f(x) = \sin\frac{1}{x} \quad \text{for } x \neq 0, \qquad f(0) = 0.$$

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$

c) Find the nth differential coefficient of

$$y = \sin^5 x \cos^3 x.$$

2.

a) Write the statement of Leibnitz's Theorem, and differentiate n times the following differential equation with respect to x

$$(1 - x^2)y_2 - xy_1 + a^2y = 0.$$

b) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0.$$

c) Discuss the differentiability at $x = 0$ of the following function:

$$f(x) = x\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \quad \text{for } x \neq 0, \qquad f(0) = 0.$$

P.T.O.

3.

a) Evaluate

$$\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{(x^2)}.$$

b) State Lagrange's Mean Value Theorem and use it to show that

$$\frac{x}{1+x}<\log(1+x)<x,\ \ x>0.$$

c) State Rolle's Theorem and discuss the applicability of Rolle's Theorem for the function

$$f(x)=|x|,\quad x\in[-1,1].$$

4.

a) In the Cauchy's Mean Value Theorem, consider $f(x)=1/x^2$ and $g(x)=1/x$ in the interval $[a,b]$, then show that c is the harmonic mean of a and b; where $a,b,c\in(-\infty,\infty)$.

b) Evaluate

$$\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x.$$

c) State Taylor's Theorem with Lagrange's form of remainder. Prove that

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^{n-1}}{(n-1)!}+\frac{x^n e^{\theta x}}{n!}$$

Where $0<\theta<1$.

5.

a) Find the asymptotes of

$$y^3+x^2y+2xy^2-y+1=0.$$

b) Find the double points and points of inflexion, if any, of the curve

$$\frac{a^2}{x^2}-\frac{b^2}{y^2}=1.$$

c) Evaluate

$$\int_0^{\pi/8}\cos^3 4x\,dx.$$

6.

a) By using reduction formula, evaluate

$$\int_0^1 x^2(1-x^2)^{3/2}\,dx.$$

b) Trace the curve

$$y^2(a+x)=x^2(3a-x).$$

c) Show that the curve

$$x^4-2ax^2y-axy^2+a^2y^2=0$$

has a cusp of the second kind at the origin.