S. No. - 10274

Roll Number:.....................

Unique Paper Code : 2223510031

Name of the Paper : Adv. Mathematical Physics
(DSE Paper)

Name of the Course : B.Sc. Prog.– (Physics)_NEP: UGCF-2022

Semester : VII

Duration : 3 hours

Maximum Marks : 90 Marks

Instructions for Students:

Attempt any five questions in total. Question No. 1 is compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculators is permitted.

Q.1. Attempt any six of the following (3x6=18)

(a) Show that the contraction of two cartesian tensors lowered the rank by any even numbers or 2.

(b) If $A_{\lambda\mu\nu} = A_{\mu\lambda\nu}$ then show that symmetry properties of a tensor are invariant.

(c) Prove the identity $\epsilon_{ilm}\epsilon_{jlm} = 2\delta_{ij}$.

(d) Find expression for gradient of a tensor.

(e) Prove that divergence of curl of a tensor is zero.

(f) What are the generators of a cyclic group G= $\langle a \rangle$ of order 10.

(g) List all the elements of A_4, where A_4 denotes the set of all even permutations in S_4.

Q.2.(a) Show that in 3-dimensional space alternating tensor has 27 components. Further shows that it is an anti-symmetric tensor of order 3. (10)

(b) What is skew symmetric tensor? Determine how many numbers of different nonzero components in skew symmetric tensor. If we assume ω_{ij} is skew-symmetric tensor of order two, with indices i and j running from 1 to 50. Calculate the number of independent components. (8)

Q.3.(a) What are equal tensors? If $\omega = a_{ij}A_iA_j$. Show that we can always write

$\omega = b_{ji}A_iA_j$, where, b_{ji} is symmetric tensor? (9)

(b) $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = [\vec{A}.(\vec{B} \times \vec{D})]\,\vec{C} - [A.(\vec{B} \times \vec{C})]\vec{D}$ (9)

Q.4. By using Cartesian tensor prove that

(a) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}$ (9)

(b) $\nabla \times (\vec{A} \times \vec{B}) \doteq (\vec{B}.\nabla)\vec{A} - (\vec{A}.\nabla)\vec{B} + \vec{A}(\nabla.\vec{B}) - \vec{B}(\nabla.\vec{A})$ (9)

Q.5.(a) Obtain an expression for moment of inertia tensor. What is rank of this tensor and prove that it is symmetric tensor. (9)

(b) If there is a mapping from G(of part 4(a)) to C*(group of non-zero complex numbers with respect to multiplication) given by

$$f: G \to C^* \text{ such that } f\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = a + ib,$$

then prove that this mapping is an isomorphism. (9)

Q.6.(a) From a group table of a multiplicative group G_1 generated from two square matrices A and B , each of order n, subject to the relations

$$A^2=B^2=(AB)^2=E$$

Where, E is the identity matrix of order n. (6)

(b) Consider a set $G_2= \{1,-1,I,-i \}$, prove that G_2 form a group w.r.t. multiplication and check if G_1 and G_2 can be isomorphic to each other or not. (6)

(c) Consider a cyclic group of order 4, $G_3= \{e,a,a^2,a^3\}$. If G_3 has 4 classes, find its irreducible representations. (6)