10268

Name of the Course : **B.Sc. Hons Physics NEP: UGCF-2022**

Semester : **VII**

Name of the Paper : **Advanced Mathematical Physics II (DSE)**

Unique Paper Code : **2223010031**

S. No. of Question Paper :

Duration: **3 hours** Maximum marks: **90**

Note: Attempt five questions in all with question no. 1 as compulsory.

1. Attempt any **six** questions. **($6 \times 3 = 18$)**

(a) If A_{ph}^{km} is a mixed tensor of rank 4, prove that A_{pm}^{km} is a mixed tensor of rank 2.

(b) Given a vector $\vec{A} = (2, 6, -3)$. Find the matrix elements of the second order skew-symmetric tensor associated with it.

(c) Show that gradient of a scalar is a covariant tensor of order one.

(d) Write down equations for two intersecting lines passing through a common point and having direction cosines l_i and m_i. Hence determine an expression for angle between the lines.

(e) Prove that if $g = g_{ij}$ is a covariant tensor of order two, and $U = U^i$ and $V = V^i$ are contravariant vectors, then the double inner product $gUV = g_{ij}U^iV^j$ is an invariant.

(f) Show that there is no distinction between contravariant and covariant vectors when we restrict to rectangular cartesian transformation of coordinates.

(g) Any second order tensor can be written as a sum of symmetric S_{ij} and an antisymmetric tensor A_{ij}. Find the corresponding symmetric and antisymmetric tensors associated with $B_{ij} = \begin{bmatrix} 6 & 2 & 8 \\ 4 & 5 & 9 \\ 1 & 3 & 0 \end{bmatrix}$.

2.

(a) Prove the following vector identities using cartesian tensors. **(12)**

i. $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = \vec{C}(\vec{A}.\vec{B} \times \vec{D}) - \vec{D}(\vec{A}.\vec{B} \times \vec{C})$

ii. $\vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{A}.\vec{B}) - (\vec{B}.\vec{\nabla})\vec{A} - (\vec{A}.\vec{\nabla})\vec{B})$

(b) Define isotropic tensors and show that the only isotropic tensor of rank one is the null vector.

(6)

3.

(a) A covariant tensor has components $(xy,\ 2y - z^2,\ xz)$ in rectangular coordinates. Find its covariant components in cylindrical coordinates. **(10)**

(b) Using the covariant components in cylindrical coordinates obtained in part (a), find the contravariant components of the tensor in cylindrical coordinates. **(8)**

4.

(a) In a Cartesian coordinate system, components of a tensor T_{ij} are given by

$$T_{ij} = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

where a, b and c are constants. Find the components of this tensor in a new cartesian coordinate which is obtained by rotating the coordinate system about the z-axis by a fixed angle θ.

(10)

(b) Prove that $\epsilon_{abc}\epsilon_{aps} = \delta_{bp}\delta_{cs} - \delta_{bs}\delta_{cp}$.
Hence find $\epsilon_{abc}\epsilon_{abs}$.

(8)

5.

(a) Given that the stress tensor P_{ij} and strain tensor e_{kl} are both symmetric tensors. Prove that rank four elastic tensor C_{ijkl} must be symmetric in its first pair of indices ($C_{ijkl} = C_{jikl}$), its last pair of indices ($C_{ijkl} = C_{ijlk}$), and also symmetric under the exchange of these pairs ($C_{ijkl} = C_{klij}$). Assume the existence of an elastic potential energy density, $U = {}^1\!/_2\, C_{ijkl}e_{ij}e_{kl}$.

(10)

(b) A body undergoes a deformation described by the displacement vector

$$\vec{u} = (kx_1{}^2, 2kx_1x_2, 0)$$

Find the strain tensor e_{ij}.

(8)

6.

(a) Check whether the Christoffel symbols of the second kind transform like a tensor or not.

(10)

(b) Prove that the divergence of a contravariant vector of rank one, A^p is

$$div\, A^p = A^p_{;p} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^k}(\sqrt{g}A^k)$$

where g is the determinant of the metric tensor g_{ij} and $A^p_{;q}$ corresponds to the covariant derivative of A^p with respect to x^q.

Use $\Gamma^p_{pr} = \frac{\partial}{\partial x^r}\ln(\sqrt{g})$.

(8)