[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 10370 **K**

Unique Paper Code : 2373570006

Name of the Paper : DSE: Introduction to Statistical Linear Models

Name of the Course : **B.A. / B.Sc. (P) under NEP-UGCF**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all, selecting **Three** from each Section.
3. Attempt **all** parts of question in continuation.
4. Use of non-programmable scientific calculator is allowed.

SECTION I

1. **(a)** Explain the general linear model and discuss the special cases arising there from.

   (b) Consider three independent random variables Y_1, Y_2, and Y_3 having common variance σ^2 and expectations as given below: $E(Y_1) = \beta_1 + \beta_3$, $E(Y_2) = \beta_1 + \beta_2$, $E(Y_3) = \beta_1 + \beta_3$. Obtain any solution of the normal equations and also obtain unbiased estimate of error variance σ^2. **(7, 8)**

2. **(a)** For a given model $Y_{n\times 1} = X_{n\times p}\,\beta_{p\times 1} + \epsilon_{n\times 1}$ with $E(\epsilon) = 0$, $V(\epsilon) = \sigma^2 I$ and $R(X) = p < n$, prove that the least squares estimator of β is BLUE.

   (b) For a linear model, the normal equations are

$$\begin{pmatrix} 10 & -2 & -8 \\ -2 & 5 & -3 \\ -8 & -3 & 11 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 16 \\ -28 \end{pmatrix}$$

   (i) Obtain any solution of the normal equations.

   (ii) When $\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ is estimable? **(7, 8)**

3. **(a)** Let $Y \sim N(\mu, I_n)$ and let A_1 and A_2 be symmetric idempotent matrices. Prove that if $Y'A_1Y$ and $Y'A_2Y$ are distributed independently as chi-square with n_1 and n_2 d.f. respectively then $A_1A_2 = 0$.

   (b) Let $Y \sim N_3(0, I)$ and let $A = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

   (i) Find the distribution of $Y'AY$ and $Y'BY$.

   (ii) Are $Y'AY$ and $Y'BY$ independent? **(7, 8)**

4. (a) Consider the models $E(Y_1) = \beta_1 + \beta_2$, $E(Y_2) = \beta_1 - \beta_2$, and $E(Y_3) = \beta_1 + 2\beta_2$, with usual assumptions. Obtain the BLUE of the unknown parameters and its variance.

(b) Let $Y \sim N(\mu, \Sigma)$, where $\mu' = (1\ \ 2\ \ 3)$ and $\Sigma = I$. Let $A = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

(i) Find expected value of $Y'AY$.

(ii) Does $Y'AY$ have a chi-square distribution? **(7, 8)**

SECTION II

5. (a) Consider the simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$ with usual assumptions. Find the distribution of $\hat{\beta}_0$ and $\hat{\beta}_1$.

(b) Consider the simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$ with usual assumptions.

(i) Obtain a test for $H_0: \beta_1 = 0$ versus $H_1: \beta_1 \neq 0$.

(ii) Obtain a confidence interval of for β_0. **(8, 7)**

6. (a) What do you mean by bias in regression estimates? Suppose we postulate the model $E(Y) = \beta_0 + \beta_1 X$ but the true model is $E(Y) = \beta_0 + \beta_1 X + \beta_2 X^2$. Calculate the biases in the least squares estimators of β_0 and β_1 by taking observations at $X = -1, 0, 1$.

(b) Define polynomials regression models. Explain the role of orthogonal polynomials in fitting polynomials in one variable. **(9, 6)**

7. (a) Give the fixed effects mathematical model for two-way classification with $m\ (> 1)$ observations per cell, stating clearly the assumptions involved. Also obtain the estimates of the parameters in the model.

(b) Derive the expected value of mean sum of squares due to errors and treatments in two-way classification with one observation per cell. Also, show that under the truth of null hypothesis, the mean sum of squares due to treatments gives an unbiased estimate of error variance. **(7, 8)**

8. Write short notes on:

(i) Multiple linear regression models

(ii) Orthogonal columns in X matrix

(iii) Lack of fit test **(5, 5, 5)**

(1000)