5. (a) Define a response surface design. Obtain the estimates of the unknown parameters of a first-order model, using the principle of least squares. Obtain their variances and covariances.

(b) Construct a CCD and a BBD for three factors and 6 centre points, and list all design points. Compare the two designs in terms of the number of observations and rotatability. (9,9)

6. (a) The following parameter estimates were calculated from multiple observations collected at the points of a {3, 2} simplex-lattice.

$b_1 = 5.5, b_2 = 7.0, b_3 = 8.0, \frac{1}{4}b_{12} = 8.75, \frac{1}{4}b_{13} = 2.25,$

$\frac{1}{4}b_{23} = 4.5$

What were the values of the average responses at the lattice points that produced the parameter estimates above?

(b) Consider the {3.2} simplex-lattice augmented with a point at the centroid $x_1 = x_1, = x_1 = \frac{1}{3}$ of the triangle. Show that when the special cubic model is fitted to the expected responses at the points of the design, $\beta_{123} = 27\eta_{123} - 12(\eta_{12} + \eta_{13} + \eta 23) + 3(\eta_1 + \eta_2 + \eta_3)$ where η_{123} is expected response at the centroid. (9,9)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 10367 **K**

Unique Paper Code : 2373010016

Name of the Paper : DSE 5b-Advanced Design of Experiments

Name of the Course : **B.Sc. (Hons.) Statistics under NEP-UGCF**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions carry equal marks.
3. Attempt any **five** questions.
4. Use of a non-programmable calculator is allowed.

1. (a) Distinguish between fixed and random effects models. For a balanced one-way classification random-effects model $y_{ij} = \mu + a_i + \varepsilon_{ij}$; i = 1,2,..., p; j = 1,2,..., n show that the sum of squares between classes and sum of squares within classes are independently distributed.

(b) A researcher wants to study the effect of two factors A and B on the output of a product. The table of observations is given below:

A1			A2		
B1	**B2**	**B3**	**B1**	**B2**	**B3**
8	7	10	5	9	6
9	6		6	8	7
	8		7	5	

Fill in the missing entries in the ANOVA table:

Sources of Variations	**DF**	**SS**	**MS**	**F-ratio**
A				
B		1.157		
Interaction AB			7.859	
Error			0.875	
Total				

(9,9)

2. A chemical engineer studies how the catalyst type affects reaction time. Testing 4 catalysts with raw material batches as blocks. Each batch allows 3 catalysts to be tested in a randomised, incomplete block design The balanced incomplete block design and observations are as follows.

Catalyst		1	2	3	4
	1	73	74		
	2		75	67	72
	3	73	75	68	
	4	75		72	75

Carry out the complete intra-block analysis of the data from this experiment (use $\alpha = 0.05$) and draw' conclusions. (18)

3. (a) What do you understand by a complete set of Mutually Orthogonal Latin Squares? Generate the elements of GF(4) and construct the complete set of MOLS of order 4.

(b) Use orthogonal Latin squares of size 5 to construct the BIBD with

$v = s^2,\ b = s^2 + s,\ r = 5 + 1,\ k = s,\ \lambda = 1$

(9.9)

4. (a) Let $v = b = 7$, $r = k = 3$. Construct a BIBD with these parameters using the method of differences. Obtain the derived design for the resultant BIBO and write its parameters.

(b) Construct a BIBD by taking (1, 3, 4, 5, 9) as the initial block under modulo 11. What are the parameters of the resultant design? Write its incidence matrix. (9,9)

P.T.O.