[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 10366 **K**

Unique Paper Code : 2373010015 (NEP-UGCF)

Name of the Paper : Financial Statistics (DSE)

Name of the Course : **BSc. (Hons.)**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **TWO** sections in this paper.
3. **All** sections are compulsory.
4. Use of non-programmable scientific calculator is allowed.

Section I (Attempt Any FIVE parts)

1. (a) What is the difference between IRR criteria and NPV. Calculate IRR and NPV for the following cashflow stream assuming interest rate is 5%.

C_0	C_1	C_2
-100	80	80

(b) What is the effective annual interest in the following situations:

(i) A savings account with 4% annual interest compounded **annually**.

(ii) A savings account with 4% annual interest compounded **monthly**.

P.T.O.

(c) ₹12000 is placed in an account at 4% compounded daily for 2 years. It is then withdrawn at the end of 2 years and placed in another bank at the rate of 5% compounded daily at 4 years. What is the balance in the second account after 4 years?

(d) Consider a long forward contract to buy a stock which has a price of S_t at time t. Let K be the delivery price and T be the maturity date. Further $V(S_t, \tau)$ denotes the value of the long forward contract at time t and τ is the time to maturity. Assume that 'r' is the interest rates during the time to maturity and is constant. If the stock pays a continuous dividend yield at rate d and involves carrying costs b, then show that

$$V(S_t, \tau) = S_t\, e^{(b-r)\tau} - Ke^{-r\,\tau}.$$

(e) An investor combines a long call and a long put with same delivery price K with C_0 and P_0 as call and put option price respectively. Plot the corresponding payoff and profit functions.

(f) Let $\{W_t; t \geq 0\}$ be a Wiener process. Is $X_t = exp\left(\sigma W_t - \frac{\sigma^2}{2}\, t\right)$ a martingale with respect to the natural filtration, where σ is a positive constant?

(g) A bank sold 3000 Calls and 2500 Puts with the same maturity and exercise price on the same underlying stock. Δ Call is 0.42 and Δ Put is -0.38. How many stocks should the company sell or buy in order to have a delta neutral position?

(5×3)

Section II (Attempt Any FIVE questions)

2. (a) Suppose that a person deposits money in a bank that pays interest at a nominal rate of 10% per year. How long will it take for his money to (i) double (ii) triple; and (iii) quadruple, if the interest is compounded continuously?

(b) Calculate the present value of each of cashflow using a discount rate of 7% and identify the most preferred cashflow stream:

(i) Cashflow A: receive \$60 today and then receive \$60 in four years.

(ii) Cashflow B: receive \$12 every year forever stating today.

(iii) Cashflow C: pay \$550 every year for five years.

(iv) Cashflow D: receive \$9 every other year with first payment being next year.

(7, 8)

3. (a) An individual who plans to retire in 20 years has decided to put an amount A in the bank at the beginning of each of the next 240 months, after which s/he will withdraw \$1,000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be?

(b) Define a geometric random walk $\{X_t;\ t \geq 0\}$. Show that for a recombining process for large t, $logX_t$ is approximately lognormally distributed.

(7, 8)

4. (a) For two European puts with the same maturity date T and delivery prices $K_1 \leq K_2$, show that at time $t < T$:

$$0 \leq P_{K_2,T}(S_t, \tau) - P_{K_1,T}(S_t, \tau) \leq (K_2 - K_1)e^{-r\tau}$$

where τ denotes the time to maturity and r is the interest rate.

(b) Consider the option combination where a trader buys a put and a call with a different strike prices and same expiration date. The put strike price K_1 is less than the call strike price K_2. Draw the corresponding payoff table and the payoff graph.

(7, 8)

5. (a) Define the following: (i) Filtration, (ii) Martingale and (iii) The Wiener process. Further, for the standard Wiener process $\{W_t; t \geq 0\}$, Calculate $Cov(2W_t, W_s - 4W_t)$ and $Cov(2W_s, 3W_s - 4W_t)$ for $0 \leq s \leq t$.

(b) Find the dynamics of $Y_t = \cos(W_t)$ for a Brownian motion W_t.

(7, 8)

6. (a) Consider the process $dS_t = \mu\, dt + \sigma\, dW_t$. Find the dynamics of the process $Y_t = g(S_t, t)$, where $g(S_t, t) = 2 + t + exp(S_t)$.

(b) What is meant by the gamma of an option position? For a put based on a stock whose price process is an Itô-process, show in usual notations that

$$rP = \Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma.$$

(7, 8)

7. (a) Mrs. Ying Chen already has a portfolio with $\Delta = -300$ and $\Gamma = 250$. She wants to make her portfolio Δ and Γ neutral. In the market, she can buy/sell stocks and call options to achieve this. She calculated the Δ and Γ of the call option as 0.55 and 1.25 respectively. What should Mrs. Chen do in order to realize her goal?

(b) Let the current price of a stock is 50. If the risk-free rate is 10% and volatility associated with the stock price movement is 20%, find the price of a call in 3 months if the exercise price is 53. Use put call parity to determine the price of a put on this stock.

Table of Φ(z) or CDF of N[0,1] values

z	-0.50	-0.40	-0.38	-0.30	-0.28	-0.20	-0.10	0.10
Φ(z)	0.3085	0.3445	0.3519	0.3820	0.3897	0.4207	0.4601	0.5398

(7, 8)

(1000)