

10267

Name of the Course : B.Sc. Hons Physics NEP: UGCF-2022
 Semester : VII
 Name of the Paper : Advanced Mathematical Physics II (DSE)
 Unique Paper Code : 2223010031
 S. No. of Question Paper :

Duration: 3 hours

Maximum marks: 90

Note: Attempt five questions in all with question no. 1 as compulsory.

1. Attempt any six questions. (6 × 3 = 18)

- (a) If A_{ph}^{km} is a mixed tensor of rank 4, prove that A_{pm}^{km} is a mixed tensor of rank 2.
- (b) Given a vector $\vec{A} = (2, 6, -3)$. Find the matrix elements of the second order skew-symmetric tensor associated with it.
- (c) Show that gradient of a scalar is a covariant tensor of order one.
- (d) Write down equations for two intersecting lines passing through a common point and having direction cosines l_i and m_i . Hence determine an expression for angle between the lines.
- (e) Prove that if $g = g_{ij}$ is a covariant tensor of order two, and $U = U^i$ and $V = V^i$ are contravariant vectors, then the double inner product $gUV = g_{ij}U^iV^j$ is an invariant.
- (f) Show that there is no distinction between contravariant and covariant vectors when we restrict to rectangular cartesian transformation of coordinates.
- (g) Any second order tensor can be written as a sum of symmetric S_{ij} and an antisymmetric tensor A_{ij} . Find the corresponding symmetric and antisymmetric tensors associated with $B_{ij} = \begin{bmatrix} 6 & 2 & 8 \\ 4 & 5 & 9 \\ 1 & 3 & 0 \end{bmatrix}$.

2.

- (a) Prove the following vector identities using cartesian tensors. (12)

- i. $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = \vec{C}(\vec{A} \cdot \vec{B} \times \vec{D}) - \vec{D}(\vec{A} \cdot \vec{B} \times \vec{C})$
- ii. $\vec{A} \times (\vec{V} \times \vec{B}) + \vec{B} \times (\vec{V} \times \vec{A}) = \vec{V}(\vec{A} \cdot \vec{B}) - (\vec{B} \cdot \vec{V})\vec{A} - (\vec{A} \cdot \vec{V})\vec{B}$

(b) Define isotropic tensors and show that the only isotropic tensor of rank one is the null vector. (6)

3.

(a) A covariant tensor has components $(xy, 2y - z^2, xz)$ in rectangular coordinates. Find its covariant components in cylindrical coordinates. (10)

(b) Using the covariant components in cylindrical coordinates obtained in part (a), find the contravariant components of the tensor in cylindrical coordinates. (8)

4.

(a) In a Cartesian coordinate system, components of a tensor T_{ij} are given by

$$T_{ij} = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$$

where a, b and c are constants. Find the components of this tensor in a new cartesian coordinate which is obtained by rotating the coordinate system about the z -axis by a fixed angle θ . (10)

(b) Prove that $\epsilon_{abc}\epsilon_{aps} = \delta_{bp}\delta_{cs} - \delta_{bs}\delta_{cp}$.

Hence find $\epsilon_{abc}\epsilon_{abs}$.

(8)

5.

(a) Given that the stress tensor P_{ij} and strain tensor e_{kl} are both symmetric tensors. Prove that rank four elastic tensor C_{ijkl} must be symmetric in its first pair of indices ($C_{ijkl} = C_{jikl}$), its last pair of indices ($C_{ijkl} = C_{ijlk}$), and also symmetric under the exchange of these pairs ($C_{ijkl} = C_{klij}$). Assume the existence of an elastic potential energy density, $U = \frac{1}{2} C_{ijkl} e_{ij} e_{kl}$. (10)

(b) A body undergoes a deformation described by the displacement vector

$$\vec{u} = (kx_1^2, 2kx_1x_2, 0)$$

Find the strain tensor e_{ij} .

(8)

6.

(a) Check whether the Christoffel symbols of the second kind transform like a tensor or not. (10)

(b) Prove that the divergence of a contravariant vector of rank one, A^p is

$$\text{div } A^p = A^p_{;p} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k} (\sqrt{g} A^k)$$

where g is the determinant of the metric tensor g_{ij} and $A^p_{;q}$ corresponds to the covariant derivative of A^p with respect to x^q .

Use $\Gamma^p_{pr} = \frac{\partial}{\partial x^r} \ln(\sqrt{g})$. (8)