10268A

Name of the Course	:	**B.Sc. (Hons.) Physics NEP: UGCF-2022**
Semester	:	**VII**
Name of the Paper	:	**Advanced Mathematical Physics (DSE)**
Unique Paper Code	:	**2223010032**
S. No. of Question Paper	:	

Duration: **3 Hours** Maximum marks: **90**

Attempt five questions in all with question no. 1 as compulsory.

1. Attempt any **six** questions: **$(6 \times 3 = 18)$**

(a) Find

$$L[\int_0^t \frac{\sin u}{u} du]$$

(b) Is the permutation $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 2\,3\,4\,1\,6\,5\,8\,7 \end{pmatrix}$ even or odd?

(c) What are the generators of the cyclic group $G = <a>$ of order 10?

(d) Find

$$L^{-1}\{\frac{3s+7}{s^2-2s-3}\}$$

(e) Show that the set of orthogonal matrices

$$G = \left\{\begin{bmatrix} \cos\propto & -\sin\propto \\ \sin\propto & \cos\propto \end{bmatrix}, \propto \in R\right\}$$

forms a SO(2) group under matrix multiplication.

(f) Construct Green's function for the inhomogeneous equation

$$y''(x) = f(x)$$

with the boundary conditions $y(0) = 0$ and $y'(1) = 0$.

(g) Show that Green's function is symmetric with respect to $\vec{r}_1$ and $\vec{r}_2$ i.e.

$$G(\vec{r}_1, \vec{r}_2) = G(\vec{r}_2, \vec{r}_1)$$

2.

(a) Solve the following differential equation using Laplace transform **(10)**

$$y'' - y = e^{2t}$$

with initial conditions $y(0) = 1$ and $y'(0) = 0$.

(b) Find Laplace transform of the following function. **(8)**

$$f(t) = e^{-4t}\frac{\sin t}{t}$$

3.

(a) Plot the function

$$f(t) = \sin t, 0 < t < \pi$$
$$= 0, \pi < t < 2\pi$$

extended periodically with period 2π. Find L[f(t)].

(10)

(b) Using convolution theorem of Laplace transforms, evaluate. **(8)**

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2 + a^2)(s^2 + b^2)}\right\}; a^2 \neq b^2$$

4.

(a) Consider a group $G = \{e, a, b, c\}$ under a binary operation "*"having Cayley group table:

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

Show that following map is isomorphic where $F: G \to S_4$

F(e) = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$, F(a) = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, F(b) = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and F(c) = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

(10)

(b) Show that the general form of the elements of $SU(2)$ group with respect to matrix multiplication is given by

$$\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$$

Also find all the generators of $SU(2)$ group. **(8)**

5.

(a) Prove that the symmetry operations of an equilateral triangle form a dihedral group D_3. Write the (2×2) matrix representation of all the elements of D_3. Show that the symmetry operations and the group of representation matrices are isomorphic to each other. **(10)**

(b) Consider the cyclic permutations generated by (1234) in Permutation/Symmetric group, S_4. Check whether these cyclic permutations form a subgroup of S_4. **(8)**

6.

(a) A pendulum in the earth gravitational field is driven by an external time dependent force F(t). If the driven y(t) from the equilibrium position remains small, we can model this system by a harmonic oscillator whose differential equation is given by

$$\ddot{y}(t) + \omega^2 y(t) = F(t), t > 0$$

with homogeneous initial conditions $y(0) = \dot{y}(0) = 0$. By using Green's function approach, solve the differential equation. **(10)**

(b) Obtain the Green's function for the linear operator

$$L(x(y)) = \left(\frac{d}{dy}\left(y\frac{d}{dy} \right) - \frac{m^2}{y} \right) x(y)$$

for x(1) = 0 and finite at x(0). m be the constant. **(8)**