(c) Consider the differential equation $t\dot{x} = 2x$. Show that $x = Ct^2$ is a solution for all the choices of constant C.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : **5151** **K**

Unique Paper Code : 2922102302

Name of the Paper : Mathematics for Business Economics-II

Name of the Course : **B.A (Hons.) Business Economics, 2025**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions. Choice is available within each question.
3. Use of simple calculator and present value tables is permitted.

1. Attempt any **four** questions : (4×7=28)

(a) Find the first and second order partial derivatives for :

(i) $z = e^{yx^2} + 3x^3y^2 - 4y^4$

(ii) Find $\frac{dy}{dx}$ if $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

(b) For each of the following functions, find the domain.

(i) $f(x,y) = \sqrt{10 - x^2 - y^2}$

(ii) $f(x,y) = \ln(xy - 1) + e^{x^2y} - y^8$

For part (i) above draw a set of three level curves.

$y \geq 0$. Suppose profits are Rs 3 per unit for product A and Rs. 5 per unit for product B. Find the production schedule (i.e. the values of x and y) that maximizes the total profit.

4. Attempt any **two** of the following : (2×7=14)

(a) Find the solution to the following difference equations :

(i) $x_1 = 2x_{t-1} + 4, \quad x_0 = 1$

(ii) $x_t - x_{t-1} + 3 = 0, \quad x_0 = 3$

(b) Show that $y = (x+1) - \frac{e^x}{3}$ is a solution to the differential equation $\frac{dy}{dx} = y - x$ which satisfies the initial condition $y = \frac{2}{3}$.

(e) Define Homogeneous and Homothetic functions. Examine which of the following function is/are homogenous and/or homothetic:

(i) $H(x, y) = \ln x + \ln y$

(ii) $G(x,y) = \sqrt{xy}\, ln\left(\frac{x^2+y^2}{xy}\right)$

3. Answer any **two** of the following : (2×8=16)

(a) Find the global maximum and minimum value of the function $f(x, y) = xy^2$ on the region R consisting of those points (x, y) with $x^2 + y^2 \leq 4$ and $x \geq 0$, $y \geq 0$.

(b) Find the maximum and minimum values of f(x, y) = 2x +3y subject to the constraint $x^2 + 4y^2 = 100$.

(c) A firm makes x units of product A and y units of product B and has a production possibilities curve given by the equation $x^2 + 25y^2 = 25000$ for $x \geq 0$,

(c) (i) Find the tangent plane to the following surfaces at the indicated points :

$z = (y - x^2)(y - 2x^2)$ at (1,3,2).

(ii) Let $f(x, y) = 2xe^{3y}$. Compute the following f(4,0) and f(1, ln2)

(d) Given that $f(x, y) = xy$; $x = 1 - \sqrt{t}$; $y = 1 + \sqrt{t}$. Find the elasticity of f with respect to t.

(e) Let $z = f(x, y)$ be given implicitly by $e^z + yz = x + y$.

(i) Find the differential dz.

(ii) Use linear approximation at the point (1,0) to approximate f(0.99, 0.01).

2. Attempt any **four** questions : (4×8=32)

(a) Find and classify the critical points of the following functions :

(i) $(x, y) = x^2 + 4y^2 - 8x - 16y + 50$

(ii) $(x, y) = x^3 + y^3 - 3axy$

(b) Define convex sets with examples. Decide which of the following are convex sets by drawing each in the xy- plane :

(i) $\{(x,y): xy \geq 1,\ x > 0,\ y > 0\}$

(ii) $\{(x,y): x^2 + y^2 > 9\}$

(c) (i) The demand functions for two products are given below, p1, p2, q1, and q2 are the prices (in dollars) and quantities for products 1 and 2.

$q_1 = 60 - 3p_1 - 6p_2$ and $q_2 = 50 - 4p_1 - p_2^2$

Are these two products complementary goods or substitute goods? What is the quantity demanded for each when the price for product 1 is Rs 2 per item and the price for product 2 is Rs 3 per item?

(ii) Examine convexity/concavity of the following function :

$$f(x, y) = x + y - e^x - e^{x+y}$$

(d) Suppose a firm has a production $X = [aL^4 + bK^4]^{½}$ $0<a<1$, $0<b<1$. where L and K are labour and capital respectively.

(i) What is the nature of returns to scale?

(ii) Verify Euler's Theorem.

(iii) Calculate the elasticity of substitution of the production function.