[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3352 **K**

Unique Paper Code : 32371301

Name of the Paper : Sampling Distributions

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question no. **1** is compulsory and attempt **two** questions from each Section.

1. Attempt any five parts:

(i) Obtain the cumulative distribution function of the first order statistic $X_{(1)}$ in a random sample of size n from a continuous population with cumulative distribution function F(x). Hence find the probability density function of $X_{(1)}$.

(ii) Discuss the limiting form of t-distribution.

(iii) Define convergence with probability one and convergence in probability. State the relationship between them.

(iv) If X has a F distribution with m, n degrees of freedom , then find the distribution of $\frac{1}{X}$. State one use of the result.

(v) Define Sampling distribution and standard error of a statistic. Obtain standard error of mean when population is large.

(vi) If $X \sim U(0,1)$, then show that $-2\,log_e X \sim \chi^2_2$. [5*3=15]

SECTION I

2.(a) Let $X_1, X_2, \ldots, X_n$ be a random sample from U(0,1) distribution. Find the coefficient of correlation between $X_{(1)}$ and $X_{(n)}$.

(b) If X is a random variable and $E(X^2) < \infty$, then prove that, $P[|X| \geq a] \leq \frac{E[X^2]}{a^2}$ for all a>0. Use Chebychev's inequality to show that for n >36, the probability that in n throws of a fair die, the number of sixes lies between $\frac{n}{6} - \sqrt{n}$ and $\frac{n}{6} + \sqrt{n}$ is at least $\frac{31}{36}$. [7, 8]

P.T.O.

3.(a) Let $X_1, X_2, \dots, X_n$ be iid random variables and $S_n = X_1 + X_2 + \cdots + X_n$. Obtain the limiting distribution of S_n when $n \to \infty$.

(b) Explain the following terms with illustrations:

(i) Type I and Type II errors,

(ii) Level of significance and critical region

(iii) p- value.

(iv) Null hypothesis and alternative hypothesis. [7, 8]

4.(a) Examine if W.L.L.N. holds for the sequence $\{S_n\}$, where $S_n = X_1 + X_2 + \cdots + X_n$. $\{X_k\}$ are i.i.d. random variables with mean μ and variance σ^2 (finite).

(b) For a random sample of size n from a continuous population whose p.d.f. is symmetric at $X = \mu$ Show that $f_r(\mu + x) = f_{n-r+1}(\mu - x)$, where $f_r(.)$is the p.d.f. of $X_{(r)}$. [8, 7]

SECTION II

5.(a) Define Student's -t and Fisher's -t statistics. Derive the sampling distribution of Fisher's -t statistic With n d.f.

(b) For a chi-square distribution with n degrees of freedom, prove that $\mu_{r+1} = 2r(\mu_r + n\mu_{r-1}), r \geq 1$. Hence find β_1 and β_2. [7, 8]

6.(a) Define F-statistic. Prove that if $n_1 = n_2$, then the median of F –distribution is at F= 1 and that the quartiles Q_1 and Q_3satisfy the condition$Q_1 Q_3 = 1$.

(b) Explain the term 'standard error and sampling distribution'. Show that in a series of n independent Bernoulli trials with constant probability of success P, the standard error of proportion of successes is $\sqrt{\frac{PQ}{n}}$, where $Q = 1 - P$. [7, 8]

7.(a) In a 2×3 contingency table, if $N = x + y + z$, $N' = x' + y' + z'$ and $N = N'$, then show that:

$$\chi^2 = \frac{(x - x')^2}{x + x'} + \frac{(y - y')^2}{y + y'} + \frac{(z - z')^2}{z + z'} \sim \chi_2^2.$$

(b) Find the mean deviation about the mean of t-distribution with n degrees of freedom.

(c) State and prove the relation between t and F distributions [5. 5. 5]