Sl. No. of Ques. Paper	:	3350
Unique Paper Code	:	32221301
Name of the Paper	:	Mathematical Physics-II
Name of the Course	:	B. Sc. (H) Physics
Semester	:	III
Duration	:	3 hours
Maximum Marks	:	75

Attempt __five__ questions in all.
***Q. No. 1** is compulsory.*

1. Attempt any ***five*** questions: (5 × 3 = 15)

(a) Given $f(x+2\pi)=f(x)$ and

$$f(x)=\begin{cases}\pi+x, & -\pi\le x\le 0\\ \pi-x, & 0\le x\le\pi\end{cases}$$

Plot $f(x)$ in the interval $(-3\pi, 3\pi)$.

(b) Determine whether the functions $\sin 2x$ and $\cos x$ are orthogonal or not in the interval $(0, 2\pi)$.

(c) Evaluate: $\int_0^{2\pi}\sin^4\theta\cos^6\theta\, d\theta$.

(d) Find the value of: $2\Gamma\left(\frac{1}{2}\right)+\Gamma\left(\frac{-1}{2}\right)$.

(e) Show that: $P_n(1)=1$.

(f) If n is an integer, prove that

$$J_{-n}(x)=(-1)^n J_n(x).$$

(g) Discuss whether $x=1$ is an ordinary, regular or irregular singular point of the following differential equation:

$$(x^2-1)y''+xy'-y=0$$

(h) Using the method of separation of variables, prove that

$$u(x,y)=Ae^{c\left(\frac{x}{2}+\frac{y}{3}\right)};\ A>0,\ c>0$$

is the solution of the following partial differential equation:

$$2\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}=0$$

2. (a) Consider a periodic function $h(x)$ such that $h(x+2\pi)=h(x)$ and

$$h(x)=\begin{cases}x, & 0\le x\le\pi\\ 2\pi-x, & \pi\le x\le 2\pi\end{cases}$$

(i) Plot $h(x)$ in the range $(-2\pi, 4\pi)$. (3)

(ii) Find the Fourier Series expansion of $h(x)$. (7)

(b) Evaluate: $\int_0^{\infty}3^{-4z^2}\,dz$ (5)

3. Consider a periodic function $f(x)$ of period 2π such that

$$f(x)=\pi-x,\ 0<x<\pi$$

(a) Plot even extension of $f(x)$ in the range $(-3\pi, 3\pi)$. (3)

(b) Find its half-range Fourier Cosine Series. (7)

(c) Show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots = \frac{\pi^2}{8}$ (5)

4. Consider the following differential equation:

$$x(x-1)y'' + (3x-1)y' + y = 0$$

(a) Find whether $x = 0$ is an ordinary, regular or irregular singular point. (3)

(b) Using Frobenius method, determine the roots of indicial equation and hence find the first solution. (4, 5)

(c) Also, find the second solution. (3)

5. (a) Prove that Rodrigue's formula for Legendre polynomials is given by

$$P_n(x) = \frac{1}{n!\,2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$$ (10)

(b) Find the value of:

$$\int_{-1}^{1} x^2 P_2(x)\, dx$$ (5)

6. (a) Prove that $J_n(x)$ is the coefficient of t^n in the expansion of $e^{\frac{x}{2}\left(t - \frac{1}{t}\right)}$. (5)

(b) Using the generating function for Bessel function or otherwise, prove that

$$J_{n+1}(x) + J_{n-1}(x) = \frac{2n}{x} J_n(x)$$ (10)

7. Using Laplace equation in 2-D, determine the steady state temperature distribution of a thin rectangular plate bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$ assuming that the edges $x = 0$, $x = a$ and $y = 0$ are maintained at zero temperature and edge $y = b$ is maintained at a steady state temperature $F(x)$. (15)

8. Using the method of separation of variables, find the general solution of 2-D wave equation for the case of rectangular membrane (area = $a \times b$):

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2};\ c > 0$$

subject to the conditions: (15)

$u(0, y, t) = u(a, y, t) = 0$, $u(x, 0, t) = u(x, b, t) = 0$ and $u(x, y, 0) = u_o(x, y)$.