$$Minimize\ z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^+ + P_4 d_4^+ + 4P_5 d_5^- + 5P_5 d_6^-$$

Subject to

$$x_1 + 2x_2 + d_1^- - d_1^+ = 40$$

$$40x_1 + 50x_2 + d_2^- - d_2^+ = 1600$$

$$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$$

$$d_1^+ + d_4^- - d_4^+ = 10$$

$$x_1 + d_5^- - d_5^+ = 30$$

$$x_2 + d_6^- - d_6^+ = 20$$

$$x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_5^+, d_6^-, d_6^+ \geq 0$$

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2911 **K**

Unique Paper Code : 42364303

Name of the Paper : Optimization Techniques

Name of the Course : **B.Sc. (Programme) Mathematical Science / B.A. (Programme) CBCS-LOCF**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. **All** question carries equal marks.
4. Use of non-programmable calculator is allowed.

1. (a) What do you mean by definiteness of the quadratic form? Write the following quadratic expression in X^TQX form and test definiteness

$f(x_1, x_2, x_3) = x_1^2 - 7x_2^2 + 5x_3^2 + 4x_1x_2 + 2x_1x_3 - 6x_2x_3$. (7)

(b) Define Convex and Concave functions and show graphically how they determine Maxima and Minima of a given function. Check whether the following functions are convex, concave or neither.

(i) $f(x) = 2x_1^2 + 6x_2^2$

(ii) $f(x) = 2x_1^2 + x_2^2 + 3x_1 + 4x_2 - 2x_1x_2$.

(8)

2. (a) Let $X_1 = (-1,0,2)$, $X_2 = (0,1,2)$ and $X_3 = (0,1,1)$. Is $X = (-1/2, 1/2, 7/4)$ a convex linear combination (clc) of X_1, X_2, X_3? Find the values of constants also. (7)

P3: Avoid overtime operation of the plant beyond 10 hours.

P4: Achieve sales goal of 10 desks, 10 tables, and 12 chairs.

P5: Minimize overtime operation as much as possible.

7. (a) Define the following terms : (7)

(i) Deviational variables

(ii) Pre-emptive priority factors

(b) Solve the following goal programming problem graphically : (8)

ABC Furniture's produce three products - tables, desks, and chairs. The furniture is produced in the central plant. The production of the desk requires 3 hours in the plant, the table 2 hours and the chair only 1 hour. The regular plant capacity is 40 hours a week. According to the marketing department, the maximum number of desks, tables and chairs that can be sold weekly are 10, 10 and 12, respectively. The president of the firm has established the following goals, according to their importance:

P1: Avoid any underutilization of production capacity.

P2: Meet the order of XYZ Store for seven desks and five chairs.

(b) Using the concept of Hessian matrix, determine the relative maximum and minimum (if any) of the following function

$$f(x) = 4x_1^2 + 3x_2^2 + x_3^2 - 6x_1x_2 + x_1x_3 - 8x_1 - 12x_3 + 5.$$

(8)

3. (a) Define the following terms (7)

(i) Constrained optimization problems

(ii) Bordered Hessian matrix.

(b) Find the critical points by Lagrange multiplier method, Hessian matrix of Lagrange function and nature of Hessian matrix at one of the critical points for the following problem

Min $f(X) = 2x_1 + x_2$

Subject to the constraint :

$$x_1^2 + x_2^2 = 25$$

$$x_1, x_2 \geq 0.$$ (8)

P.T.O.

4. (a) Explain non-linear programming problem with examples. Give a mathematical formulation for the general non-linear programming problem. (7)

(b) Solve the following non-linear programming problem using the Kuhn-Tucker conditions

Min $f(X) = 2x_1 - x_1^2 + 3x_2 - 2x_2^2$

Subject to the constraints :

$$x_1 + 3x_2 \leq 6$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0.$$ (8)

5. (a) Describe a quadratic programming problem. How is it different from non-linear programming problem? (7)

(b) Solve the following quadratic programming problem using Beale's method. (8)

Max $f(X) = -10x_1^2 - x_2^2 + 10x_1 + 25x_2 - 4x_1x_2$

Subject to the constraints :

$$x_1 + x_2 \leq 9$$

$$x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0.$$

6. (a) What is goal programming? What are the different types of goal programming problems, describe them briefly? (7)

(b) Formulate the following problem as a goal programming problem. (8)