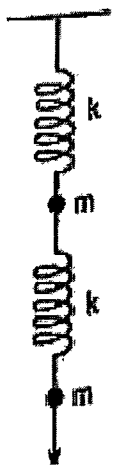
(b) Two identical springs, each having a spring constant k, are suspended vertically from a rigid horizontal support. Equal masses m are attached to the lower ends of the springs, as shown in the figure.

(i) Taking only vertical motion, derive the equations of motion of the system.

(ii) Determine the normal mode frequencies of oscillation for the system. Using the normal modes explain the motion of the masses of the system. (8)



[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5774 **K**

Unique Paper Code : 2222014701

Name of the Paper : Classical Mechanics

Name of the Course : **B.Sc. Hons. – (Physics)_ NEP: UGCF-2022**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory.
3. Attempt any **four** out of remaining **five** questions.
4. **All** questions carry equal marks.
5. Symbols have usual meaning unless specified.

P.T.O.

1. Attempt any **six** parts out of the following :

(a) A particle of mass m moves under a central force such that its trajectory is given by, $r = r_0 e^{\theta}$. Determine the expression for the force that produces this spiral orbit. (Here, r is the radial distance of the particle from the centre of force, r_0 is a constant that sets the size scale of the orbit, θ is the angle the radius vector makes with a fixed direction.)

(b) Identify the constraint and determine its nature (holonomic or non-holonomic) for the following

(i) A pendulum of length l swinging in a vertical plane.

(ii) A particle free to move anywhere within a spherical region.

(c) Determine the Hamiltonian corresponding to the Lagrangian given below. Here ω and α are constants.

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}\omega^2 q^2 - \alpha q^3.$$

5. (a) Derive the Euler's equations for a rigid body in the absence of external torque and obtain the equation representing the inertia ellipsoid. (10)

(b) Define the inertia tensor and explain how its diagonalization leads to principal moments. Show that the cross-terms vanish when the coordinate axes coincide with the principal axes. (8)

6. (a) Consider a double pendulum with masses m_1 and m_2; and lengths l_1 and l_2.

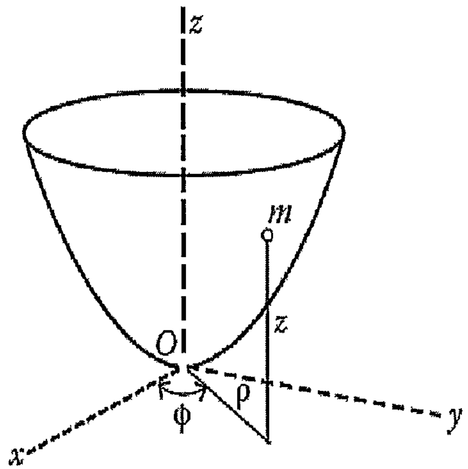
(i) Derive the equations of motion for small oscillations.

(ii) Determine the normal mode frequencies and the corresponding eigenvectors.

(iii) Discuss the special cases: $m_1 >> m_2$, $m_2 >> m_1$ and $m_1 = m_2$ (10)

(i) Show that the angular momentum of the particle about the axis of symmetry of the system is conserved.

(ii) Determine the equation of motion of the particle along the z-axis.



(10)

(b) State Bertrand's theorem and show that only inverse-square and Hookes-law forces produce closed orbits. (8)

(d) For the transformation, $Q = aq + bp$, $P = cq + dp$, find the condition under which it is canonical and determine the relationship among a, b, c, and d.

(e) List three difference between Newtonian and Lagrangian mechanics.

(f) An alpha particle with an energy of 7.7 MeV is scattered by a lead nucleus (Pb, Z = 82, A = 207). If the angle of scattering is 30°, determine the scattering cross-section and the impact parameter for this scattering event.

(g) Describe the concept of a 3D rotation matrix.

(3*6)

2. (a) A simple pendulum of length l and mass m is attached to a pivot that slides freely without friction along a horizontal wire. Taking x (the horizontal position of the pivot) and θ (the angle of pendulum with the vertical) as generalized coordinates,

(i) Write the Lagrangian of the system.

(ii) Derive the Euler-Lagrange equations of motion for both the generalized coordinates.

(iii) Linearize the equations for small oscillations ($\theta \ll 1$) and find the coupled linear equations. (10)

(b) A particle of mass m is attached to a rigid, massless rod of length l and moves under gravity, forming a spherical pendulum. The motion is constrained to the surface of a sphere.

(i) Write down the Lagrangian $L(\theta, \phi, \dot{\theta}, \dot{\phi})$ of the system in spherical coordinates (θ, ϕ) where θ is the polar angle and ϕ is the azimuthal angle.

(ii) Derive the Euler-Lagrange equations of motion for both θ and ϕ. (8)

3. (a) A charged particle of charge q and mass m, is moving in an electromagnetic field, such that its Lagrangian is given below. (Here, ϕ and $\vec{A}$ are

the electrodynamic potentials of the field.)

$$L = \frac{1}{2}mv^2 - q\phi + q\vec{A}\cdot\vec{v}$$

(i) Find Hamiltonian of the system.

(ii) Determine Hamilton's equation of motion.

(iii) Show that equations determined in part (ii) are consistent with Lorentz force law. (10)

(b) Prove that the total time rate of change of phase point density ρ vanishes in phase space.

$$\frac{d\rho}{dt} + \frac{\partial\rho}{\partial t} + [\rho, H] = 0 \qquad (8)$$

4. (a) A particle of mass m is constrained to move without friction on the inner surface of a paraboloid of revolution defined by, $x^2 + y^2 = az^2$, where a, is a positive constant. The particle is under the influence of gravity.