SL No of QP	:	**5755**
Name of the Department	:	Physics
Name of the Course	:	B.Sc. Hons.–(Physics)_NEP: UGCF-2022
Name of the Paper	:	Mathematical Physics-I
Semester	:	I
Unique Paper Code	:	2222011101
Question Paper Set No	:	1

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in total.
3. Question No. **1** is compulsory.
4. **All** questions carry equal marks.

Q1. Answer any six of the following: **[6x3 = 18]**

(a) A temperature field in a room is given by $T_{(x,y,z)} = 20 - x^2 - 2y^2 - z^2$ (in °C). Find the direction of maximum temperature increase at the point (1, -1, 2). What is the rate of increase in that direction?

(b) Expand the function $f(x) = \ln(1+x)$ about x=0 up to the 4th order. Approximate $f(x = 0.5)$.

(c) Solve: $(2y^3 + 2)\,dx + 3xy^2 dy = 0$

(d) Find $\iint \vec{F}.\hat{n}\ dS$ for the vector field $\vec{F}_{(x,y,z)} = z\,\hat{\imath} + y\,\hat{\jmath} + x\,\hat{k}$, over the unit sphere $x^2 + y^2 + z^2 = 1$.

(e) Given the vectors $\vec{a} = (3,4)$ and $\vec{b} = (5,-2)$, compute their dot product before and after rotating the axes counterclockwise by 45°, and verify invariance.

(f) Find the Wronskian of the set $\{sin\, x, 2\, cos\, x, 3\, sinx + cosx\}$, and determine whether the set is linearly dependent or independent.

(g) From the probability distribution

X:	1	2	3	4	5
P(X):	k	2k	3k	2k	k

Find the value of k, and evaluate E(X).

Q2. (a) Using index notation δ_{ij} and ε_{ijk}, prove that $\vec{a}\ X\ (\vec{b}\ X\ \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ **8**

(b) If $\vec{A}$ is an irrotational vector field, show that $\vec{A} X \vec{r}$ is solenoidal, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. **5**

(c) Evaluate: $\nabla.(r^3\vec{r})$ **5**

Q3. (a) Solve by the Method of Variation of Parameters: **9**

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{(1+e^{-x})}$$

(b) Solve the differential equation: **9**

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3 log\, x^2, \ (x > 0)$$

Q4. (a) Solve by Method of Undetermined Coefficients: **9**

$$\frac{d^2y}{dx^2} + 4y = 2\sin x \cos x - 7$$

(b) A resistor of $R = 2000\,\Omega$ and a capacitor of capacitance $C = 5 \times 10^{-6}\,F$ are connected in series to a 100 Volts DC source. What is the current $I(t)$ as a function of time for this RC charging circuit, given that $I = 10^{-2}\,A$ at the time the switch is closed $(t = 0)$. Also, determine the initial charge on the capacitor at $t = 0$. **9**

Q5. (a) Verify Green's theorem in the plane for $\oint_c (xy + y^2)\,dx + x^2\,dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. **9**

(b) Consider the scalar function $T(x, y, z) = z^2$ defined on the tetrahedral domain enclosed by the three coordinate planes and the plane $x + y + z = 1$. The vertices of this tetrahedron are (0,0,0), (1,0,0), (0,1,0), and (0,0,1). Compute the volume integral of T over this region. **9**

Q6. (a) Obtain the expression for the mean and variance of the Binomial distribution. **8**

(b) In a radioactive decay process, the number of decays per minute is Poisson-distributed with a mean of 4. Find the probability that exactly 3 decays occur in one minute. **5**

(c) Find the scalar potential function ϕ for $\vec{F} = y^2\hat{\imath} + 2xy\,\hat{\jmath} - z^2\hat{k}$. **5**