[This question paper contains 2 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5753 **K**

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt **two** parts from each question.

1. (a) (i) Solve the equation

$$x^3 - 14x^2 - 84x + 216 = 0,$$

given that the roots are in geometrical progression.

(ii) Find an upper limit to the roots of the equation $x^4 - 5x^3 + 7x^2 - 8x + 1 = 0$. (4.5+3)

(b) Find all the integral roots of $x^3 - 5x^2 - 2x + 24 = 0$. (7.5)

(c) Find all the rational roots of $32y^3 - 6y - 1 = 0$. (7.5)

2. (a) Find the value of z, where

$$z = \frac{(1-i)^{10}\left(\sqrt{3}+i\right)^5}{\left(-1-i\sqrt{3}\right)^{10}}$$ (7.5)

(b) Find the fourth roots of $z = \sqrt{3}+i$ and represent them geometrically to show that they lie on a circle of radius $2^{1/4}$. (7.5)

(c) Solve the equation

$$z^{10} + (-2 + i)z^5 - 2i = 0.$$ (7.5)

3. (a) Solve

$$y^3 - 15y - 126 = 0$$

using Cardan's method. (7.5)

P.T.O.

(b) Prove that $n^2 - 2$ is never divisible by 4. (7.5)

(c) Let $a = qb + r$ where $a, b, q, r \in \mathbb{Z}$. Show that $\gcd(a,b) = \gcd(b, r)$. Find the gcd of 143 and 481. (7.5)

4\. (a) (i) Let $a, b, c \in \mathbb{Z}$ be such that $a|bc$. If a, b are relatively prime, show that $a|c$.

(ii) Find the gcd of $a = 1575$, $b = 231$ and represent it in the form $ma + nb$ for suitable integers m and n. (4+3.5)

(b) Solve the following pair of congruences. If no solution exists, explain why.

$$x + 5y \equiv 3 \pmod 9$$
$$4x + 5y \equiv 1 \pmod 9$$
(7.5)

(c) Let $a \equiv x \pmod n$ and $b \equiv y \pmod n$. Then, show that

(i) $a + b = (x + y)(\text{mod } n)$

(ii) $a - b = (x - y)(\text{mod } n)$

(iii) $ab = (xy)(\text{mod } n)$ (2.5+2.5+2.5)

5\. (a) Define a group. Is Z_5 under addition module 5, a group? Justify your answer. (2+5.5)

(b) Let $GL(2, \mathbb{R}) = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a,b,c,d \in \mathbb{R}, |A| \neq 0\right\}$ be the set of all 2×2 matrices over $\mathbb{R}$ with nonzero determinant. Is $GL(2, \mathbb{R})$ a group with respect to matrix multiplication? Is it Abelian? Justify your answer. (5+1+1.5)

(c) Define the Centre $Z(G)$ of a group G. Is $Z(G)$ a subgroup of G? Is it Abelian? Give reasons to support your answer. (5+2.5)

6\. (a) Let $G = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a,b,c,d \in \mathbb{Z}\right\}$ be a group under addition and $H \subset G$ be such that $H = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G : a+b+c+d = 0\right\}$. Prove that H is a subgroup of G. What if 0 is replaced by 1? (5+2.5)

(b) Define inverse of an element in a group. If G is a group where every element is its own inverse, show that G is commutative. (2+5.5)

(c) Define a cyclic group. List down all subgroups of Z_{10}. Are these subgroups cyclic? What are their generators? What is the order of each subgroup? (1+2+1+2+1.5)