[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 13449 **K**

Unique Paper Code : 2354000015

Name of the Paper : Topics In Multivariate Calculus

Name of the Course : **Common Prog Group**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) Define continuity of a function of two variables. Show that f is continuous at $(0,0)$,where

$$f(x,y)=\begin{cases} y\ sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0 & , x=0 \end{cases}.$$

(b) (i) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$ does not exist.

(ii) Compute the slope of the tangent line to the graph of f at the given point P_0 in the direction parallel to the xz –plane and yz –plane, where $f(x,y)=x^2\ sin(x+y)$ and $P_0\left(\frac{\pi}{2},\frac{\pi}{2},0\right)$.

(c) At a certain factory, the daily output is $Q=60\ K^{\frac{1}{2}}L^{\frac{1}{3}}$ units, where K denotes the capital investment (in units of $1000) and L the size of the labor force (in worker-hours). The current capital investment is $900,000 and 1000 worker-hours of labor are used each day. Estimate the change in output that will result if capital investment is increased by $1000 and labor is decreased by 2 worker-hours.

2. (a) Define differentiability of a function of two variables. Give an example of a non-differentiable function for which f_x and f_y exist.

(b) (i) Let $f(x,y,z)=xy\ sin(xz)$. Find ∇f_0 at the point $P_0(1,-2,\pi)$ and then compute the directional derivative of f at P_0 in the direction of the vector $v=-2i+3j-5k$.

(ii) Find an equation for each horizontal tangent plane to the surface

$$z=5-x^2-y^2+4y.$$

(c) Given that the largest and smallest values of $f(x,y)=1-x^2-y^2$ subject to the constraint $x+y=1$ with $x\geq 0$ and $y\geq 0$ exist, use the method of Lagrange multipliers to find the extrema.

3. (a) (i) Evaluate $\iint_R 2xe^y\, dA$; $R: -1 \le x \le 0,\ 0 \le y \le ln2$ using iterated integration.

(ii) Find volume of the solid bounded below by the rectangle R in the xy plane and above by the graph of $z = f(x,y)$ where $f(x,y) = \sqrt{xy}$ $R:\ 0 \le x \le 1,\ 0 \le y \le 4$.

(b) Evaluate $\iint_D (2y - x)dA$; D is the region between the parabola $y = x^2$ and the line $y = 2x$.

(c) Compute the area in polar form of the region D bounded above by the line $y = x$ and below by the circle $x^2 + y^2 - 2y = 0$.

4. (a) Evaluate $\iiint_D x dV$ where D is the solid in the first octant bounded by the cylinder $x^2 + y^2 = 4$ and the plane $2y + z = 4$.

(b) Using cylindrical coordinates, find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half cone $z = \sqrt{x^2 + y^2}$ and xy-plane.

(c) Let D be the region in xy plane that is bounded by coordinate axes and the line $x + y = 1$. Use suitable change of variables $u = x - y,\ v = x + y$ to compute the integral $\iint_D \left(\frac{x-y}{x+y}\right)^4 dy\, dx$.

5(a) Evaluate the line integral $\int_C (-y\, dx + x\, dy + xzdz)$, where C is the helical path given by $x = cost, y = sint, z = t$ for $0 \le t \le 2\pi$.

(b) Verify the vector field $\mathbf{F}(x,y) = (y - x^2)\boldsymbol{i} + (x + y^2)\mathbf{j}$ is conservative and find a scalar potential f for $\mathbf{F}$. Then evaluate the line integral $\int_C \mathbf{F}.d\mathbf{R}$, where C is any smooth path connecting $A(0,0)$ to $B(1,1)$.

(c) Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has area πab using the line integral.

6(a) Evaluate $\iint_S 2x dS$ where S is the portion of the plane $x + y + z = 1$ with $x \ge 0, y \ge 0$, $z \ge 0$.

(b) Use Stokes' theorem to evaluate $\iint_S (\text{curl } \mathbf{F}.\mathbf{N})\, dS$, where $\mathbf{F} = x\,\boldsymbol{i} + y^2\,\boldsymbol{j} + xyz\,\boldsymbol{k}$ and S is that part of the paraboloid $z = 1 - x^2 - y^2$ with $z \ge 0$. Use the upward unit normal vector.

(c) Use the divergence theorem to evaluate the surface integral $\iint_S \mathbf{F}.\mathbf{N}dS$ for the $\mathbf{F} = (x^2+y^2 - z^2)\,\mathbf{i} + x^2y\,\mathbf{j} + 3z\,\mathbf{k}$ and S is the surface of the upper five faces of the unit cube $0 \le x \le 1,\ 0 \le y \le 1, 0 \le z \le 1$ missing $z = 0$.