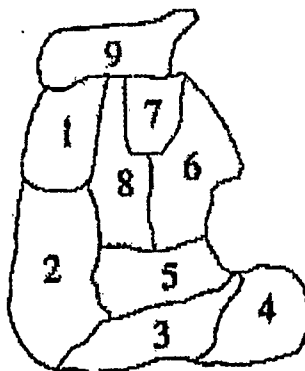


- (c) Draw a graph corresponding to the map shown at the right and find a coloring which requires the least number of colors. What is the chromatic number of the graph?



[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 13448 K

Unique Paper Code : 2354000014

Name of the Paper : Introduction to Graph Theory

Name of the Course : GE

Semester : VII

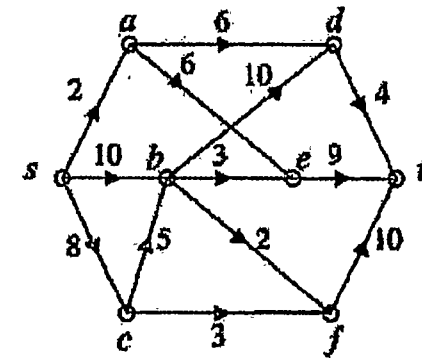
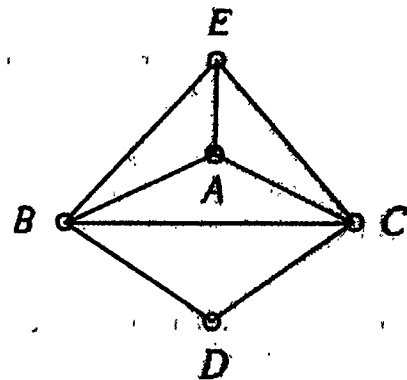
Duration : 3 Hours

Maximum Marks : 90

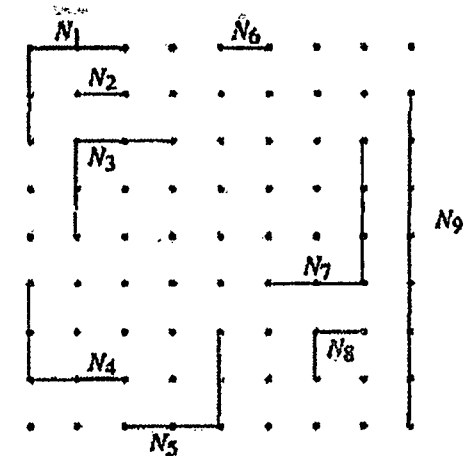
Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **Both** parts of a question to be attempted together.
4. **All** questions carry equal marks (7.5 marks each).

1. (a) Is it possible for a graph to have degree sequence 4, 4, 3, 3, 2, 2, 2, 1? Draw a graph with 5 Vertices v_1, v_2, v_3, v_4, v_5 such that $\deg v_1=3$, v_2 is an odd vertex, $\deg v_3=2$ and v_4 & v_5 are adjacent.
- (b) Define Hamiltonian Path and Euler Path. Consider the graph given below. Is it Hamiltonian or Eulerian? Is there a Hamiltonian cycle?

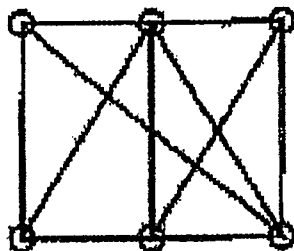


- (b) Draw the associated line-of-sight graph G .



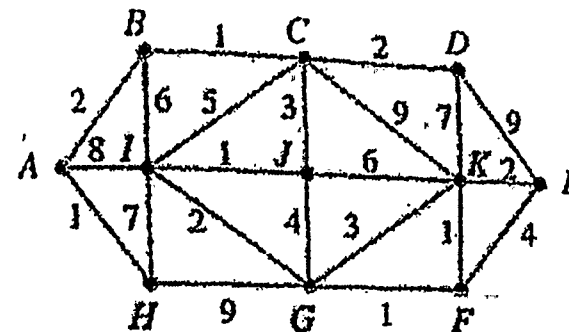
(b) Let G be a connected planar graph with p vertices and q edges, where $p \geq 3$. Then $q \geq 3p - 6$.

(c) Label the regions defined by your plane graph and list the edges which form the boundary of each region.

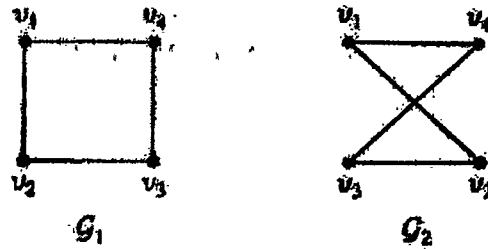


6. (a) Find a maximum flow in the directed network shown.

(c) Use Dijkstra's Algorithm to find shortest path from Vertex A to E in the given weighted Graph.



2. (a) Find the Adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a Permutation matrix P such that $A_2 = PA_1P^T$. Prove that G_1 and G_2 are Isomorphic.

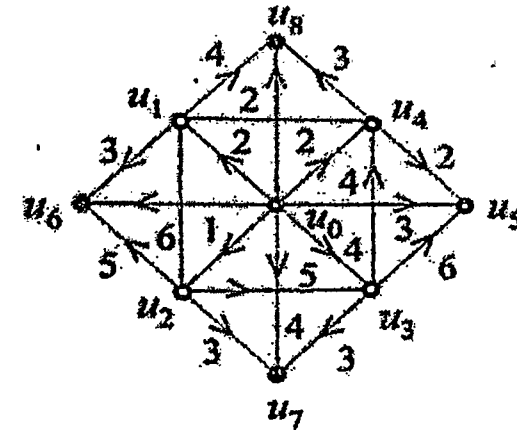


(b) Solve the following Traveling Salesman Problem.

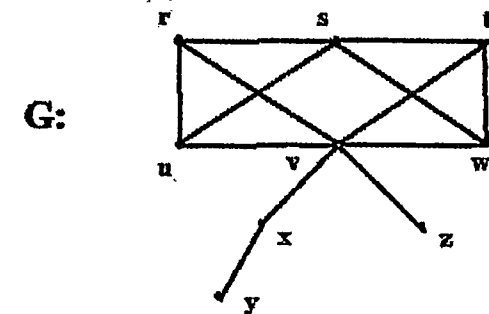
The distances from one city to another city are given in the following table :

	A	B	C	D	E
A	-	13	19	16	15
B	14	-	18	15	16
C	14	18	-	14	13
D	13	17	16	-	18
E	19	17	16	17	-

(c) Define a Complete Bipartite Graph? Determine whether the below graph is a Complete Bipartite or not? Explain.



5. (a) (i) Define cut-vertex of a Graph G and find all the cut-vertices of the graph G given below :

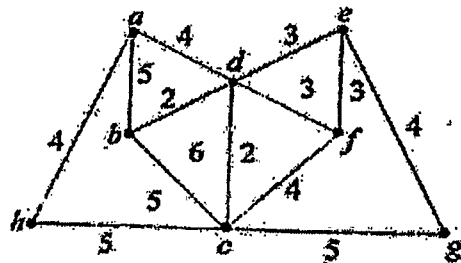


(ii) Draw the induced graph by deleting all the cut-vertices.

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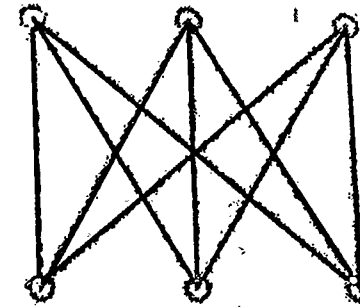
- (ii) Find the minimum total transfer time from the Emergency Department (Node 1) to the Recovery Ward (Node 6). Also, Identify the fastest patient route.
4. (a) Find minimum spanning tree of the following graph using Kruskal's algorithm.



- (b) Apply Prim's algorithm to the above given graph in (a) to find the minimum spanning tree and compare results.
- (c) Apply Bellman's algorithm to the given digraph, to find the length of the shortest path from u_0 to every other vertex.

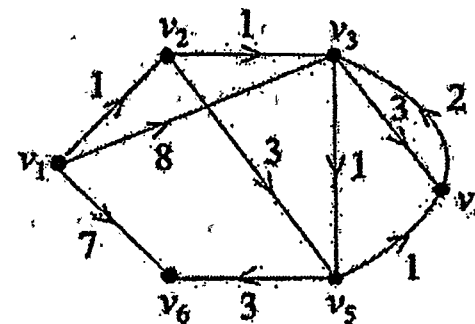
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Draw $K_{2,3}$. Is it a Bipartite or a Complete Bipartite Graph?

3. (a) Apply Bellman-Ford algorithm in the digraph given below to find the length of shortest path from v_1 to every other vertex.

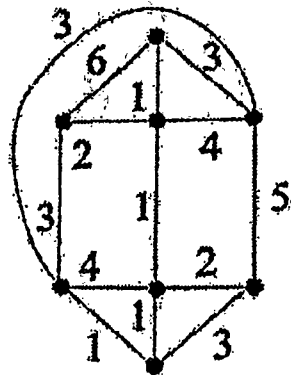


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(b) Solve the Chinese Postman problem for the given weighted graph.



(c) A hospital's central scheduling office must determine the fastest sequence of departmental transfers for a patient undergoing emergency surgery. Each transfer includes transport & preparation time (in minutes). The patient starts in the Emergency Department (Node 1) and must

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reach the Recovery Ward (Node 6) after passing through necessary units. The available paths and total transfer times are as follows:

Transfer (Node)	From Department (Node)	To Department	Time (minutes)
A	Emergency (1)	Radiology (2)	15
B	Emergency (1)	Laboratory (3)	20
C	Radiology (2)	Surgery (4)	30
D	Radiology (2)	ICU (5)	45
E	Laboratory (3)	Surgery (4)	25
F	Laboratory (3)	ICU (5)	40
G	Surgery (4)	Recovery (6)	20
H	ICU (5)	Recovery (6)	15

(i) Draw a weighted directed graph representing the hospital's patient transfer network.

P.T.O.