[This question paper contains 2 printed pages.]

Your Roll No.................

Sr. No. of Question Paper : 6273 **K**

Unique Paper Code : 2352203501

Name of the Paper : Elements of Real Analysis

Name of the Course : **B.A. / B.Sc. (P) with Mathematics as Non-Major / Minor**

Semester : V

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Show that for all x, y $\in \mathbb{R}$, $|x + y| \leq |x| + |y|$.

   (b) Let $a \geq 0$ be a fixed non-negative element in an ordered field F. Then for all x, y $\in$ F, $|x - y| < a \Leftrightarrow y - a < x < y + a$.

   (c) Define supremum of non-empty subsets of $\mathbb{R}$. Find supremum and infimum of the following sets :

   (i) $\left\{\frac{1}{x} : x > 0\right\}$

   (ii) $\left\{\sin\left(\frac{n\pi}{2}\right) : n \in \mathbb{N}\right\}$

2. (a) Prove that a set cannot have more than one greatest lower bound.

   (b) Prove that if $\{a_n\}$ is a bounded sequence and $\{b_n\}$ converges to 0, then $\{a_n b_n\}$ converges to 0.

   (c) Prove that for any fixed real number c, $\lim \frac{c^n}{n!} = 0$.

3. (a) Prove that every convergent sequence is bounded but the converse is not true.

(b) Show that the sequence $<f_n>$ defined by $f_n = \left(\sqrt{n+1} - \sqrt{n}\right)$, $n \in \mathbb{N}$ is convergent.

(c) Prove that a bounded sequence with a unique limit point is convergent.

4. (a) Show that

(i) $\lim_{n\to\infty} \frac{n}{x^n} = 0$ if $x > 1$.

(ii) If $k \in \mathbb{N}$, then $\lim_{n\to\infty} \frac{n^k}{k^n} = 0$ if $k \geq 2$.

(b) Consider the sequence $\{x_n\}$ defined inductively by $x_1 = 1$ and $\forall n \in \mathbb{N}$

$$x_{n+1} = \sqrt{x_n + 2}.$$

Show that $\{x_n\}$ converges and find its limit.

(c) Define Monotonic sequence. Prove that every monotonic bounded sequence converges.

5. (a) State Integral test for convergence of a positive term series. Use it to examine the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$.

(b) Test for convergence $1 + \frac{x^2}{2^p} + \frac{x^4}{4^p} + \frac{x^6}{6^p} + \cdots\cdots\cdots\cdots\cdots$.

(c) If $\sum_{n=1}^{\infty} a_n$ is a positive term convergent series, then show that $\sum_{n=1}^{\infty} \frac{a_n}{n}$ is convergent.

6. (a) Prove that if the series $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. Give an example of a divergent series such that $\lim_{n\to\infty} a_n = 0$. What do you conclude?

(b) Determine whether the geometric series

$$\sum_{n=0}^{\infty} \frac{3^n + 5^n}{4^n}$$

converges or diverges. Find the sum of the series if it converges.

(c) Test for convergence and absolute convergence the series

$$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + \cdots.$$