[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 7033 **K**

Unique Paper Code : 2372574701 (NEP-UGCF)

Name of the Paper : DSC: Stochastic Process and Queueing Theory

Name of the Course : **B.A. / B.Sc. (P)**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Q. No. **1** is compulsory.
4. Attempt **five** more questions from the Q. No. **2** to **7**.
5. Use of a non-programmable scientific calculator is allowed.

SECTION I

1. Attempt any **five** parts.

(a) For a (M/M/1) with SIRO service discipline and infinite system capacity queuing system, obtain the expected number of customers in the system.

(b) Let X be a non-negative integral valued random variable with $P(X=k) = p_k$; $k = 0,1,2...$ and probability generating function P(s). Find the generating function of $P(X < k)$.

(c) Let the probability of transition from a dry day to a wet day be 0.45 and that of transition from a wet day to a dry day be 0.6. The initial probability distribution of a day being a dry day or a wet day is (0.8 0.2). Find the probability of the second day being a wet day.

(d) Construct the transition probability matrix when game starts with \$8 and gambler with initial capital \$3 starts the game and his probabilities of winning, losing and drawing a game are (0.46, 0.44,. 10) respectively and stopping criteria is either gambler ruins or gambler wins all amount.

(e) Show that the initial distribution together with transition matrix completely defines a Markov chain.

(f) Let the random variable Y have a modified geometric distribûtion given by

$P[Y = k] = q^{k-1}p$, $k = 1,2,....$; obtain the p.g.f. and hence the mean of Y.

(g) Let $X(t) = A_0 + A_1t + A_2t^2$, where A_i, $i = 0,1,2$ are uncorrelated random variables with mean 0 and variance 1. Is $\{X(t), t \geq 0\}$ covariance stationary?

(3×5)

Section II

2. (a) Find the generating function of the Fibonacci numbers $\{f_n\}$ defined by $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$, and $f_0 = 0$, $f_1 = 1$.

(b) Let $S_N = X_1 + X_2 +...+ X_N$, where X_i's are i.i.d. random variables and N is a random variable independent of X_i's. Show that $Var(X_N) = E(N)$ $Var(Xi) + Var(N) (E(Xi))^2$.

(7,8)

3. (a) Consider the process $X(t) = ACoswt + BSinwt$ where A and B are uncorrelated random variables with mean 0 and variance σ^2 and w is a positive constant. Is the process $\{X(t), t \in T\}$ covariance stationary?

(b) A child can either go by a car or catch the school bus to go to his school every day. He never goes two days in a row in school bus, but if he goes by bus one day, then the next day he is equally likely to travel by car as he is to go by school bus again. On the first day of the week he throws a die and goes by bus to his school iff "5 " appears. Find the probability that

(i) he goes by car to his school on the second day of the week.

(ii) he catches a bus in the long run. (7,8)

4. (a) Consider the Markov Chain having State space S = {1,2,3,4} and transition probability matrix

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0.25 & 0.125 & 0.125 & 0.5 \end{bmatrix}$$

(i) Is the chain irreducible?

(ii) Find the nature of states of the Markov chain.

(b) Show that in an irreducible Markov chain, all the states are of the same type. (7,8)

5. (a) Let q_z be the probability of the gambler's ultimate ruin in classical ruin problem and p_z be the probability of his winning, find q_z and p_z. Also obtain the probability of gambler's ruin when a gambler plays against an infinitely rich adversary when p=q.

(b) On 1st August 2023 each of the three leading hair oil companies Dabur, Marico, HUL launched a new hair oil. When the new oils were launched each oil company Dabur, Marico, HUL had 65%, 15% and 20%market share respectively, company had an equal Dabur retained 80% of its customers but lost 10% to Marico and 10% to HUL. Marico retained 50% of its customers and lost 50% to Dabur. HUL managed to retain 60% of its customers but lost 25% to Dabur and 15% to Marico. Construct the corresponding transition probability matrix and identify the nature of states. Use Markov chain analysis to determine the market share of the three companies after two months and also predict the market share of each company in the long run. (7,8)

6. (a) State the additive property of a Poisson process. If $\{N_i(t), t \in T\}$ and $\{N_2(t), t \in T\}$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then

(i) Obtain the distribution of $N(t) = N_1(t) + N_2(t)$ and identify the same.

(ii) Find the inter-arrival distribution of $N(t)$.

(b) Define Yule-Furry process. Obtain the p.g.f. and hence the mean and variance of the process. (7,8)

7. (a) An airline organization has one reservation clerk on duty in its local branch. The clerk handles information regarding passenger reservations and flight timings. Assume that the number of customers arriving during any given period is in a Poisson fashion with an arrival rate of 8 per hour and that the reservation clerk can serve a customer in 6 minutes on an average with an exponentially distributed service time. Calculate the following:

(i) Probability that the system is busy.

(ii) Average time that a customer spends in the system.

(iii) Average queue length.

(b) For the model (M/M/1): (N/FIFO), derive the steady state equations and find the probability distribution of number of customers in the system. (7,8)