[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 7032 **K**

Unique Paper Code : 2222514701

Name of the Paper : Statistical Mechanics

Name of the Course : **B.Sc. Prog. – (Physics)_NEP: UGCF-2022**

Semester : VII

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory.
3. Attempt any **4** questions out of the remaining **5** questions.
4. Symbols have their usual meaning.

Q. 1 Answer any ***six*** from the following: (6 x 3 =18)

(i) Define the term, microstate and macrostate giving one example each?

(ii) Draw the graph for the variation of entropy (S) and specific heat (C_V) with respect to temperature ($T > 0$) for a two level system.

(iii) For a classical system of N-atoms, energy of each atom of mass m is given by

$$\varepsilon = \beta\left(x^3 + y^3\right) + \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{2}m\omega^2\left(x^2 + y^2\right) + \alpha\left(x + y\right)$$

where α, β and ω are constants. Determine the total energy and average energy per particle of the system using equipartition theorem.

(iv) Write three properties of liquid Helium.

(v) Calculate the total number of possible ways of distributing 2 bosons in 3 quantum states.

(vi) Write three properties of the particles that follows Fermi-Dirac statistics.

(vii) Calculate the Fermi energy for silver using the following data:

Atomic weight = 108, Density = 10.5 g/cm^3.

Q. 2 (a) Show that the entropy (S) and thermodynamic probability (Ω) are related as

$$S = k_B ln\Omega$$ 6

(b) A system of 4 ideal classical particles can be in any one of 4-states of energies ε, 2ε, 3ε and 4ε respectively. If total energy is 15ε for this system, calculate its entropy. 6

(c) Prove that the partition function for an ideal monoatomic gas consisting of N distinguishable particles in a cubical volume V at temperature T is given by :

$$Z = V^N(2\pi mk_BT/h^2)^{3N/2}$$ 6

Q. 3 Consider a two level system of N distinguishable particles. Each particle can occupy only one of two energy levels of energy ε_1 and ε_2 (where $\varepsilon_1 < \varepsilon_2$). Particles are distributed in such a way that n_1 particles are present in level of energy ε_1 and n_2 particles resides in energy level ε_2. (Assume N to be very large and $N = n_1 + n_2$)

(a) Find the entropy and energy of this system. Show that entropy of this system is maximum when $n_2 = N/2$. 9

(b) Obtain the general expression of temperature for the above mentioned isolated system and explain how it is possible to attain negative temperature for this system. 9

Q. 4. (a) Obtain Planck's radiation formula using Bose-Einstein statistics. 9

(b) For a photon gas, show that the specific heat at constant volume (C_V) is 3 times its entropy. $C_V = 3S$ 9

Q. 5 (a) Obtain the Bose-Einstein distribution function. Show that at high temperatures, this function reduces to Maxwell-Boltzmann distribution function. 9

(b) Show that below the transition temperature ($T < T_c$), pressure of an ideal quantum gas consisting of N non-relativistic spinless Boson particles in 3 dimensions is given by

$$P(T) = 0.51(N_{ex}k_B/V)T$$

where N_{ex} is the number of bosons in the excited state and V is the volume. 9

Q. 6 (a) Using the following data of a white dwarf star, find the nature of the electron gas inside the same: a sphere consisting of helium gas of mass M = 10^{30} kg with density $\rho = 10^{10}$ kg/m^3 and temperature T of the order of 10^6 K. (use mass of proton $\approx 10^{-27}$ kg and mass of electron $\approx 10^{-30}$ kg) 9

(b) Calculate the temperature (in terms of Fermi temperature T_F) at which the chemical potential μ of an electron gas become zero. 9

Use:

$$\int_0^\infty \frac{x^{1/2}}{e^x + 1} = 0.678$$

Some useful constants and integrals :

c = 3 x 10^8 ms^{-1},

m_e = 9.11 x 10^{-31} Kg,

b = 2898 μm K,

σ = 5.67 x 10^{-8} Jm^{-2}s^{-1}K^{-4},

h = 6.626 X 10^{-34} J s = 4.136 X 10^{-15} eV s,

k_B = 1.38 x 10^{-23} J/K = 8.617 x 10^{-5} eV/K,

$$\int_0^\infty \frac{x^{n-1}dx}{(z^{-1}e^x)-1} = \Gamma(n)g_n(z)\,;\ g_n(1) = \zeta(n)$$

P.T.O.

$\int_0^\infty \frac{x^{n-1}dx}{(z^{-1}e^x)+1} = \Gamma(n)f_n(z)\,;\ f_n(z) = g_n(z) - [2^{1-n}g_n(z^2)]$

$\zeta(3/2) = 2.612\,, \zeta(5/2) = 1.341\,, f_{3/2}(1) = 0.765\,,$

$\zeta(2) = \frac{\pi^2}{6}\,, \zeta(4) = \frac{\pi^4}{6}\,, \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$