5. (a) Eliminate the arbitrary function f from

$$z = x + y + f(xy),$$

and obtain the partial differential equation. Also specify the order.

(b) Find the general solution of the partial inferential equation

$$yu_y - xu_x = 1$$

(c) Reduce the equation

$$yu_x + u_y = x$$

into canonical form and obtain the general solution.

6. (a) Apply the method of separation of variable u(x, y) = f(x) + g(y) to solve the equation

$$u_x^2 + u_y^2 = 1$$

(b) Solve the Cauchy problem

$$yu_x + xu_y = 0, \quad u(0, y) = e^{-y^2}$$

(c) Classify the partial differential equation

$$yu_{xx} - xu_{yy} = 0, \quad x > 0, \quad y > 0.$$

Also, obtain the general solution by reducing it to canonical form.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6482 **K**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks

1. (a) Determine the constant A such that the following equation is exact, and solve the resulting exact equation :

$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right)dx + \left(\frac{1}{x^2} - \frac{1}{x}\right)dy = 0$$

(b) Solve the initial-value problem :

$2x(y+1)dx - (x^2+1)dy = 0,\ y(1) = -5$

(c) Solve the differential equation

$(2xy^2 + y)dx + (2y^3 - x)dy = 0$

2. (a) Find the orthogonal trajectories of the family of ellipses having center at the origin, a focus at the point (c, 0), and semimajor axis of length 2c.

(b) In a certain bacteria culture the rate of increase in the number of bacteria is proportional to the number present.

(i) If the number triples in 5 hours, how many will be present in 10 hours?

(ii) When will the number present be 10 times the number initially present?

(c) Show that e^{-x}, e^{3x} and e^{4x} are linearly independent solutions of the differential equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

Also, write the general solution.

3. (a) Using the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2e^{2x}$$

(b) Solve the initial value problem

$y'' - y' + 3y = 0,\quad y(0) = 6,\ y'(0) = -7$

(c) Using the method of Variation of Parameters, find the general solution of the differential equation

$$y'' + y' - 6y = \sin(2x)$$

4. (a) Find the general solution of the given differential equation

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = 4x$$

(b) Write the general solution of the 10th order homogenous differential equation whose auxiliary equation has the following roots :

3, 3, 3, –3, 5 + i, 5 –i, 0, 0, 1 + i, 1 – i.

Also, verify that e^{2x} and e^{3x} are solutions to the differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(c) Find the general solution of the linear system :

$$\frac{dx}{dt} + \frac{dy}{dt} - x = -2t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - 3x - y = t^2$$