Unique Paper Code : 2273100008

Name of the Paper : Advanced Econometrics (DSE)

Name of the Course : B.A. (Honours) Economics NEP ana B.A. Programme Economics NEP

Semester : V/VII

Duration : 3 hours

Maximum Marks : 90

Instructions for Candidates:

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answers may be written in English or Hindi but the same medium should be used throughout the paper.
3. The question paper consists of six questions.
4. **Q1 is compulsory.** It carries 10 marks.
5. **Do any four out of five questions from Q2 to Q6.** Each question carries 20 marks.
6. Use of simple non programmable calculator is allowed.
7. Statistical tables are attached for your reference

Q-1 Consider the multiple regression model:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where the CLRM assumptions MLR.1 through MLR.4 hold (linearity, random sampling, no perfect collinearity, and zero conditional mean). Suppose the data are given as:

Y = (5,7,9,6,8), X_1 = (1,0,2,1,3), X_2 = (2,1,0,3,2), X_3 = (4,5,3,2,1)

a) Write the regression model in matrix notation, clearly defining all matrices.

b) Using matrix algebra, calculate the OLS estimator $\hat{\beta} = (X'X)^{-1}X'Y$.

c) Suppose we are interested in the linear combination $\theta = \beta_1 + \beta_2$:

(i) Show that $\hat{\theta} = \hat{\beta}_1 + \hat{\beta}_2$ is an unbiased estimator of θ.

(ii) Derive Var($\hat{\theta}$) in terms of Var($\hat{\beta}_1$), Var($\hat{\beta}_2$), and Cov($\hat{\beta}_1$, $\hat{\beta}_2$).

d) Compute the Total Sum of Squares (TSS), Explained Sum of Squares (ESS), and Residual Sum of Squares (RSS) using matrix notation.

e) Find the coefficient of determination R^2 and interpret it. 5*2=10

Q-2

a) Suppose a linear probability model (LPM) yields a predicted value of Y that is equal to −0.4. Explain why this is problematic.

b) Why are the coefficients of probit and logit models estimated using the method of maximum likelihood (MLE), rather than using the instrumental variables (IV) method?

c) Your friend is analyzing survey data on individuals' exercise habits to study what predicts regular physical activity in your city. She asks if she should use a logit, probit, or linear probability model. What are the advantages and disadvantages of each model in this context? Which would you recommend, and why?

d) Now suppose the dependent variable Y_i represents the number of hours per week an individual exercises, but many individuals report zero hours. Explain why a Tobit model may be more appropriate in this case. Discuss how the Tobit model differs from the logit and probit models in both purpose and interpretation.

e) You are analyzing household spending on luxury goods, where some households report zero expenditure because they choose not to buy such goods. Describe how you could use a two-part model that combines a logit/probit model with an OLS or Tobit model to analyze this data. What does each stage of the model capture? 5*4 =20

Q-3 Suppose the market for milk can be described by the following system:

$$Q_s = \alpha_1 + \beta_1 P + \gamma_1 Z_s + u_1$$

$$Q_d = \alpha_2 + \beta_2 P + \gamma_2 Z_d + u_2$$

where Q_s and Q_d are quantity supplied and demanded, P is price, and Z_s and Z_d are exogenous supply and demand shifters respectively. In equilibrium, $Q_s = Q_d = Q$.

a) Define what is meant by a structural equation and a reduced form equation in this context.

b) Explain why OLS estimation of the supply curve generally produces biased estimates of β_1.

c) Under what condition is the demand equation identified? What role does the variable Z's play in this identification?

d) Describe how the two-stage least squares (2SLS) method would be used to estimate the demand equation in this system.

4*5=20

Q-4 Consider a school-level analysis examining the impact of health insurance coverage on healthcare utilization among high school students, modelled as follows:

$$H_s = \beta_0 + \beta_1 I_c + \beta_2 F_P + \beta_3 A_v + \beta_4 urban + u_1 \qquad \text{Equation (1)}$$

where:

H_s= percentage of students who visited a hospital in the past year,
I_c= percentage of students with health insurance coverage,
F_p= percentage of female students in the school population.
A_v = average family income at the school,
urban = dummy variable indicating whether a school is in an urban area.

The model is estimated at the school level.

a) Interpreting the preceding Equation 1 in a causal, ceteris paribus fashion, what is the expected sign of β_1? Explain the intuition behind this expected sign in the context of healthcare access and insurance coverage.

b) Why might H_s and I_c be jointly determined? Describe the potential feedback mechanisms that could create reverse causality between these two variables. In particular, discuss how healthcare utilization could influence the decision or likelihood of insurance enrolment, rather than operating only in the forward causal direction.

c) Suppose the true relationship determining insurance coverage is:

$$I_c = \gamma_0 + \gamma_1 H_s + \textit{other factors}, \qquad \text{Equation (2)}$$

and assume $\gamma_1 > 0$, meaning that higher hospital utilization rates increase the likelihood of insurance enrolment. What is the likely bias in estimating β_1 by OLS?

d) Suppose there exists a binary variable, G_p, which equals 1 if a school participates in a government or private health insurance assistance program, and 0 otherwise. Explain how G_p can be used as an instrumental variable (IV) to estimate β_1 (and the other parameters) in the Equation (1). What conditions must G_p satisfy to serve as a valid instrument in this context? Specifically, discuss its relevance and exogeneity with respect to both the hospital visits and insurance rate equations.

4*5=20

Q-5

(a) Explain the consistency and efficiency properties of the Fixed Effects (FE) estimator in panel data models. Under what conditions is the FE estimator unbiased and consistent?

(b) What is the "within transformation" used in the Fixed Effects model? Explain how it helps in eliminating unobserved individual-specific effects.

(c) What is a "balanced panel" vs. an "unbalanced panel"? How does it affect estimation?

(d) What are the limitations of using panel data models with short time periods (small T, large N)?

4*5=20

Q-6 Consider the following infinite distributed lag model:

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \cdots + \mu_t$$

a) Show how the Koyck transformation can be used to transform the distributed lag model given above into an estimable form of an AR model.

b) What restrictions does Koyck impose on the λ, where λ is the rate of decline of the distributed lag : $\beta_k = \beta_0 \lambda^k$, k=0, 1, 2, 3...

c) Write the three features of the Koyck scheme.

d) Prove that for the Koyck model, the mean lag is $\frac{\lambda}{1-\lambda}$ 4*5=20

CONTENTS

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
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LOGARITHMS

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54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
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56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
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63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
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65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
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ANTILOGARITHMS

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.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

PRESENT VALUE TABLE

This table shows the discount factor for an amount at the end of *n* period at r%

Periods	Interest rates (r)									
(n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.990	.980	.971	.962	.952	.943	.935	.926	.917	.909
2	.980	.961	.943	.925	.907	.890	.873	.857	.842	.826
3	.971	.942	.915	.889	.864	.840	.816	.794	.772	.751
4	.961	.924	.888	.855	.823	.792	.763	.735	.708	.683
5	.951	.906	.863	.822	.784	.747	.713	.681	.650	.621
6	.942	.888	.837	.790	.746	.705	.666	.630	.596	.564
7	.933	.871	.813	.760	.711	.665	.623	.583	.547	.513
8	.923	.853	.789	.731	.677	.627	.582	.540	.502	.467
9	.914	.837	.766	.703	.645	.592	.544	.500	.460	.424
10	.905	.820	.744	.676	.614	.558	.508	.463	.422	.386
11	.896	.804	.722	.650	.585	.527	.475	.429	.388	.350
12	.887	.788	.701	.625	.557	.497	.444	.397	.356	.319
13	.879	.773	.681	.601	.530	.469	.415	.368	.326	.290
14	.870	.758	.661	.577	.505	.442	.388	.340	.299	.263
15	.861	.743	.642	.555	.481	.417	.362	.315	.275	.239
16	.853	.728	.623	.534	.458	.394	.339	.292	.252	.218
17	.844	.714	.605	.513	.436	.371	.317	.270	.231	.198
18	.836	.700	.587	.494	.416	.350	.296	.250	.212	.180
19	.828	.686	.570	.475	.396	.331	.277	.232	.194	.164
20	.820	.673	.554	.456	.377	.312	.258	.215	.178	.149

Periods	Interest rates (r)									
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	.901	.893	.885	.877	.870	.862	.855	.847	.840	.833
2	.812	.797	.783	.769	.756	.743	.731	.718	.706	.694
3	.731	.712	.693	.675	.658	.641	.624	.609	.593	.579
4	.659	.636	.613	.592	.572	.552	.534	.516	.499	.482
5	.593	.567	.543	.519	.497	.476	.456	.437	.419	.402
6	.535	.507	.480	.456	.432	.410	.390	.370	.352	.335
7	.482	.452	.425	.400	.376	.354	.333	.314	.296	.279
8	.434	.404	.376	.351	.327	.305	.285	.266	.249	.233
9	.391	.361	.333	.308	.284	.263	.243	.225	.209	.194
10	.352	.322	.295	.270	.247	.227	.208	.191	.176	.162
11	.317	.287	.261	.237	.215	.195	.178	.162	.148	.135
12	.286	.257	.231	.208	.187	.168	.152	.137	.124	.112
13	.258	.229	.204	.182	.163	.145	.130	.116	.104	.093
14	.232	.205	.181	.160	.141	.125	.111	.099	.088	.078
15	.209	.183	.160	.140	.123	.108	.095	.084	.074	.065
16	.188	.163	.141	.123	.107	.093	.081	.071	.062	.054
17	.170	.146	.125	.108	.093	.080	.069	.060	.052	.045
18	.153	.130	.111	.095	.081	.069	.059	.051	.044	.038
19	.138	.116	.098	.083	.070	.060	.051	.043	.037	.031
20	.124	.104	.087	.073	.061	.051	.043	.037	.031	.026

CUMULATIVE PRESENT VALUE

This table shows the annuity factor for an amount at the end of each year for *n* years at r%.

Periods	Interest rates (r)									
(n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201
19	17.226	15.679	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365
20	18.046	16.351	14.878	13.590	12.462	11.470	10.594	9.818	9.129	8.514

Periods	Interest rates (r)									
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833
2	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528
3	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106
4	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589
5	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991
6	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326
7	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605
8	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837
9	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031
10	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192
11	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327
12	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439
13	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533
14	6.982	6.628	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611
15	7.191	6.811	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675
16	7.379	6.974	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730
17	7.549	7.120	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775
18	7.702	7.250	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812
19	7.839	7.366	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843
20	7.963	7.469	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870

FUTURE VALUE TABLE

Future value of Re. 1 i.e. $(1 + r)^n$ where r = interest rate; n = number of periods until payment or receipt.

Periods	Interest rates (r)							
(n)	1%	2%	3%	4%	5%	6%	7%	8%
1	1.01000	1.02000	1.03000	1.04000	1.05000	1.06000	1.07000	1.08000
2	1.02010	1.04040	1.06090	1.08160	1.10250	1.12360	1.14490	1.16640
3	1.03030	1.06121	1.09273	1.12486	1.15763	1.19102	1.22504	1.25971
4	1.04060	1.08243	1.12551	1.16986	1.21551	1.26248	1.31080	1.36049
5	1.05101	1.10408	1.15927	1.21665	1.27628	1.33823	1.40255	1.46933
6	1.06152	1.12616	1.19405	1.26532	1.34010	1.41852	1.50073	1.58687
7	1.07214	1.14869	1.22987	1.31593	1.40710	1.50363	1.60578	1.71382
8	1.08286	1.17166	1.26677	1.36857	1.47746	1.59385	1.71819	1.85093
9	1.09369	1.19509	1.30477	1.42331	1.55133	1.68948	1.83846	1.99900
10	1.10462	1.21899	1.34392	1.48024	1.62889	1.79085	1.96715	2.15892
11	1.11567	1.24337	1.38423	1.53945	1.71034	1.89830	2.10485	2.33164
12	1.12683	1.26824	1.42576	1.60103	1.79586	2.01220	2.25219	2.51817
13	1.13809	1.29361	1.46853	1.66507	1.88565	2.13293	2.40985	2.71962
14	1.14947	1.31948	1.51259	1.73168	1.97993	2.26090	2.57853	2.93719
15	1.16097	1.34587	1.55797	1.80094	2.07893	2.39656	2.75903	3.17217
16	1.17258	1.37279	1.60471	1.87298	2.18287	2.54035	2.95216	3.42594
17	1.18430	1.40024	1.65285	1.94790	2.29202	2.69277	3.15882	3.70002
18	1.19615	1.42825	1.70243	2.02582	2.40662	2.85434	3.37993	3.99602
19	1.20811	1.45681	1.75351	2.10685	2.52695	3.02560	3.61653	4.31570
20	1.22019	1.48595	1.80611	2.19112	2.65330	3.20714	3.86968	4.66096
21	1.23239	1.51567	1.86029	2.27877	2.78596	3.39956	4.14056	5.03383
22	1.24472	1.54598	1.91610	2.36992	2.92526	3.60354	4.43040	5.43654
23	1.25716	1.57690	1.97359	2.46472	3.07152	3.81975	4.74053	5.87146
24	1.26973	1.60844	2.03279	2.56330	3.22510	4.04893	5.07237	6.34118
25	1.28243	1.64061	2.09378	2.66584	3.38635	4.29187	5.42743	6.84848
26	1.29526	1.67342	2.15659	2.77247	3.55567	4.54938	5.80735	7.39635
27	1.30821	1.70689	2.22129	2.88337	3.73346	4.82235	6.21387	7.98806
28	1.32129	1.74102	2.28793	2.99870	3.92013	5.11169	6.64884	8.62711
29	1.33450	1.77584	2.35657	3.11865	4.11614	5.41839	7.11426	9.31727
30	1.34785	1.81136	2.42726	3.24340	4.32194	5.74349	7.61226	10.06266
31	1.36133	1.84759	2.50008	3.37313	4.53804	6.08810	8.14511	10.86767
32	1.37494	1.88454	2.57508	3.50806	4.76494	6.45339	8.71527	11.73708
33	1.38869	1.92223	2.65234	3.64838	5.00319	6.84059	9.32534	12.67605
34	1.40258	1.96068	2.73191	3.79432	5.25335	7.25103	9.97811	13.69013
35	1.41660	1.99989	2.81386	3.94609	5.51602	7.68609	10.67658	14.78534
36	1.43077	2.03989	2.89828	4.10393	5.79182	8.14725	11.42394	15.96817
37	1.44508	2.08069	2.98523	4.26809	6.08141	8.63609	12.22362	17.24563
38	1.45953	2.12230	3.07478	4.43881	6.38548	9.15425	13.07927	18.62528
39	1.47412	2.16474	3.16703	4.61637	6.70475	9.70351	13.99482	20.11530
40	1.48886	2.20804	3.26204	4.80102	7.03999	10.28572	14.97446	21.72452

FUTURE VALUE OF RE. 1 PER PERIOD PAYMENT

Future value of Re. 1 i.e. $\left|(1+r)^n - 1\right|/r$ where r = interest rate; n = number of periods until payment or receipt.

Periods	Interest rates (r)							
(n)	1%	2%	3%	4%	5%	6%	7%	8%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.01000	2.02000	2.03000	2.04000	2.05000	2.06000	2.07000	2.08000
3	3.03010	3.06040	3.09090	3.12160	3.15250	3.18360	3.21490	3.24640
4	4.06040	4.12161	4.18363	4.24646	4.31013	4.37462	4.43994	4.50611
5	5.10101	5.20404	5.30914	5.41632	5.52563	5.63709	5.75074	5.86660
6	6.15202	6.30812	6.46841	6.63298	6.80191	6.97532	7.15329	7.33593
7	7.21354	7.43428	7.66246	7.89829	8.14201	8.39384	8.65402	8.92280
8	8.28567	8.58297	8.89234	9.21423	9.54911	9.89747	10.25980	10.63663
9	9.36853	9.75463	10.15911	10.58280	11.02656	11.49132	11.97799	12.48756
10	10.46221	10.94972	11.46388	12.00611	12.57789	13.18079	13.81645	14.48656
11	11.56683	12.16872	12.80780	13.48635	14.20679	14.97164	15.78360	16.64549
12	12.68250	13.41209	14.19203	15.02581	15.91713	16.86994	17.88845	18.97713
13	13.80933	14.68033	15.61779	16.62684	17.71298	18.88214	20.14064	21.49530
14	14.94742	15.97394	17.08632	18.29191	19.59863	21.01507	22.55049	24.21492
15	16.09690	17.29342	18.59891	20.02359	21.57856	23.27597	25.12902	27.15211
16	17.25786	18.63929	20.15688	21.82453	23.65749	25.67253	27.88805	30.32428
17	18.43044	20.01207	21.76159	23.69751	25.84037	28.21288	30.84022	33.75023
18	19.61475	21.41231	23.41444	25.64541	28.13238	30.90565	33.99903	37.45024
19	20.81090	22.84056	25.11687	27.67123	30.53900	33.75999	37.37896	41.44626
20	22.01900	24.29737	26.87037	29.77808	33.06595	36.78559	40.99549	45.76196
21	23.23919	25.78332	28.67649	31.96920	35.71925	39.99273	44.86518	50.42292
22	24.47159	27.29898	30.53678	34.24797	38.50521	43.39229	49.00574	55.45676
23	25.71630	28.84496	32.45288	36.61789	41.43048	46.99583	53.43614	60.89330
24	26.97346	30.42186	34.42647	39.08260	44.50200	50.81558	58.17667	66.76476
25	28.24320	32.03030	36.45926	41.64591	47.72710	54.86451	63.24904	73.10594
26	29.52563	33.67091	38.55304	44.31174	51.11345	59.15638	68.67647	79.95442
27	30.82089	35.34432	40.70963	47.08421	54.66913	63.70577	74.48382	87.35077
28	32.12910	37.05121	42.93092	49.96758	58.40258	68.52811	80.69769	95.33883
29	33.45039	38.79223	45.21885	52.96629	62.32271	73.63980	87.34653	103.96594
30	34.78489	40.56808	47.57542	56.08494	66.43885	79.05819	94.46079	113.28321
31	36.13274	42.37944	50.00268	59.32834	70.76079	84.80168	102.07304	123.34587
32	37.49407	44.22703	52.50276	62.70147	75.29883	90.88978	110.21815	134.21354
33	38.86901	46.11157	55.07784	66.20953	80.06377	97.34316	118.93343	145.95062
34	40.25770	48.03380	57.73018	69.85791	85.06696	104.18375	128.25876	158.62667
35	41.66028	49.99448	60.46208	73.65222	90.32031	111.43478	138.23688	172.31680
36	43.07688	51.99437	63.27594	77.59831	95.83632	119.12087	148.91346	187.10215
37	44.50765	54.03425	66.17422	81.70225	101.62814	127.26812	160.33740	203.07032
38	45.95272	56.11494	69.15945	85.97034	107.70955	135.90421	172.56102	220.31595
39	47.41225	58.23724	72.23423	90.40915	114.09502	145.05846	185.64029	238.94122
40	48.88637	60.40198	75.40126	95.02552	120.79977	154.76197	199.63511	259.05652

POISSON DISTRIBUTION

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

For a given value of λ an entry indicates the probability of a specific value of x.

	λ									
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0.3679	0.1353	0.0498	0.0183	0.0067	0.0025	0.0009	0.0003	0.0001	0.0000
1	0.3679	0.2707	0.1494	0.0733	0.0337	0.0149	0.0064	0.0027	0.0011	0.0005
2	0.1839	0.2707	0.2240	0.1465	0.0842	0.0446	0.0223	0.0107	0.0050	0.0023
3	0.0613	0.1804	0.2240	0.1954	0.1404	0.0892	0.0521	0.0286	0.0150	0.0076
4	0.0153	0.0902	0.1680	0.1954	0.1755	0.1339	0.0912	0.0573	0.0337	0.0189
5	0.0031	0.0361	0.1008	0.1563	0.1755	0.1606	0.1277	0.0916	0.0607	0.0378
6	0.0005	0.0120	0.0504	0.1042	0.1462	0.1606	0.1490	0.1221	0.0911	0.0631
7	0.0001	0.0034	0.0216	0.0595	0.1044	0.1377	0.1490	0.1396	0.1171	0.0901
8	0.0000	0.0009	0.0081	0.0298	0.0653	0.1033	0.1304	0.1396	0.1318	0.1126
9		0.0002	0.0027	0.0132	0.0363	0.0688	0.1014	0.1241	0.1318	0.1251
10		0.0000	0.0008	0.0053	0.0181	0.0413	0.0710	0.0993	0.1186	0.1251
11			0.0002	0.0019	0.0082	0.0225	0.0452	0.0722	0.0970	0.1137
12			0.0001	0.0006	0.0034	0.0113	0.0263	0.0481	0.0728	0.0948
13			0.0000	0.0002	0.0013	0.0052	0.0142	0.0296	0.0504	0.0729
14				0.0001	0.0005	0.0022	0.0071	0.0169	0.0324	0.0521
15				0.0000	0.0002	0.0009	0.0033	0.0090	0.0194	0.0347
16					0.0000	0.0003	0.0014	0.0045	0.0109	0.0217
17						0.0001	0.0006	0.0021	0.0058	0.0128
18						0.0000	0.0002	0.0009	0.0029	0.0071
19							0.0001	0.0004	0.0014	0.0037
20							0.0000	0.0002	0.0006	0.0019
21								0.0001	0.0003	0.0009
22								0.0000	0.0001	0.0004
23									0.0000	0.0002
24										0.0001
25										0.0000

POISSON DISTRIBUTION

	λ									
x	11	12	13	14	15	16	17	18	19	20
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0010	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0037	0.0018	0.0008	0.0004	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.0102	0.0053	0.0027	0.0013	0.0006	0.0003	0.0001	0.0001	0.0000	0.0000
5	0.0224	0.0127	0.0070	0.0037	0.0019	0.0010	0.0005	0.0002	0.0001	0.0001
6	0.0411	0.0255	0.0152	0.0087	0.0048	0.0026	0.0014	0.0007	0.0004	0.0002
7	0.0646	0.0437	0.0281	0.0174	0.0104	0.0060	0.0034	0.0019	0.0010	0.0005
8	0.0888	0.0655	0.0457	0.0304	0.0194	0.0120	0.0072	0.0042	0.0024	0.0013
9	0.1085	0.0874	0.0661	0.0473	0.0324	0.0213	0.0135	0.0083	0.0050	0.0029
10	0.1194	0.1048	0.0859	0.0663	0.0486	0.0341	0.0230	0.0150	0.0095	0.0058
11	0.1194	0.1144	0.1015	0.0844	0.0663	0.0496	0.0355	0.0245	0.0164	0.0106
12	0.1094	0.1144	0.1099	0.0984	0.0829	0.0661	0.0504	0.0368	0.0259	0.0176
13	0.0926	0.1056	0.1099	0.1060	0.0956	0.0814	0.0658	0.0509	0.0378	0.0271
14	0.0728	0.0905	0.1021	0.1060	0.1024	0.0930	0.0800	0.0655	0.0514	0.0387
15	0.0534	0.0724	0.0885	0.0989	0.1024	0.0992	0.0906	0.0786	0.0650	0.0516
16	0.0367	0.0543	0.0719	0.0866	0.0960	0.0992	0.0963	0.0884	0.0772	0.0646
17	0.0237	0.0383	0.0550	0.0713	0.0847	0.0934	0.0963	0.0936	0.0863	0.0760
18	0.0145	0.0255	0.0397	0.0554	0.0706	0.0830	0.0909	0.0936	0.0911	0.0844
19	0.0084	0.0161	0.0272	0.0409	0.0557	0.0699	0.0814	0.0887	0.0911	0.0888
20	0.0046	0.0097	0.0177	0.0286	0.0418	0.0559	0.0692	0.0798	0.0866	0.0888
21	0.0024	0.0055	0.0109	0.0191	0.0299	0.0426	0.0560	0.0684	0.0783	0.0846
22	0.0012	0.0030	0.0065	0.0121	0.0204	0.0310	0.0433	0.0560	0.0676	0.0769
23	0.0006	0.0016	0.0037	0.0074	0.0133	0.0216	0.0320	0.0438	0.0559	0.0669
24	0.0003	0.0008	0.0020	0.0043	0.0083	0.0144	0.0226	0.0328	0.0442	0.0557
25	0.0001	0.0004	0.0010	0.0024	0.0050	0.0092	0.0154	0.0237	0.0336	0.0446

CUMULATIVE POISSON DISTRIBUTION

For a given value of λ an entry indicates the probability of a equal to or less than the specific value of x

For example if λ = 5, the probability of x being equal to or less than 6 is 0.7622 (in bold below).

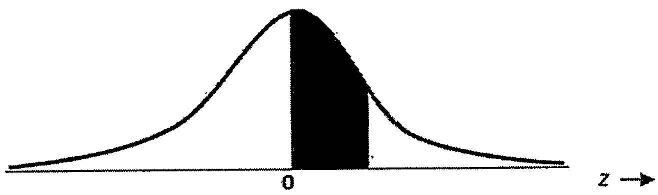
	λ									
x	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0.3679	0.1353	0.0498	0.0183	0.0067	0.0025	0.0009	0.0003	0.0001	0.0000
1	0.7358	0.4060	0.1991	0.0916	0.0404	0.0174	0.0073	0.0030	0.0012	0.0005
2	0.9197	0.6767	0.4232	0.2381	0.1247	0.0620	0.0296	0.0138	0.0062	0.0028
3	0.9810	0.8571	0.6472	0.4335	0.2650	0.1512	0.0818	0.0424	0.0212	0.0103
4	0.9963	0.9473	0.8153	0.6288	0.4405	0.2851	0.1730	0.0996	0.0550	0.0293
5	0.9994	0.9834	0.9161	0.7851	0.6160	0.4457	0.3007	0.1912	0.1157	0.0671
6	0.9999	0.9955	0.9665	0.8893	**0.7622**	0.6063	0.4497	0.3134	0.2068	0.1301
7	1.0000	0.9989	0.9881	0.9489	0.8666	0.7440	0.5987	0.4530	0.3239	0.2202
8		0.9998	0.9962	0.9786	0.9319	0.8472	0.7291	0.5925	0.4557	0.3328
9		1.0000	0.9989	0.9919	0.9682	0.9161	0.8305	0.7166	0.5874	0.4579
10			0.9997	0.9972	0.9863	0.9574	0.9015	0.8159	0.7060	0.5830
11			0.9999	0.9991	0.9945	0.9799	0.9467	0.8881	0.8030	0.6968
12			1.0000	0.9997	0.9980	0.9912	0.9730	0.9362	0.8758	0.7916
13				0.9999	0.9993	0.9964	0.9872	0.9658	0.9261	0.8645
14				1.0000	0.9998	0.9986	0.9943	0.9827	0.9585	0.9165
15					0.9999	0.9995	0.9976	0.9918	0.9780	0.9513
16					1.0000	0.9998	0.9990	0.9963	0.9889	0.9730
17						0.9999	0.9996	0.9984	0.9947	0.9857
18						1.0000	0.9999	0.9993	0.9976	0.9928
19							1.0000	0.9997	0.9989	0.9965
20								0.9999	0.9996	0.9984
21								1.0000	0.9998	0.9993
22									0.9999	0.9997
23									1.0000	0.9999
24										1.0000

CUMULATIVE POISSON DISTRIBUTION

	λ									
x	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000	0.0000
5	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002	0.0001
6	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005	0.0003
7	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015	0.0008
8	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039	0.0021
9	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089	0.0050
10	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183	0.0108
11	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347	0.0214
12	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606	0.0390
13	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984	0.0661
14	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497	0.1049
15	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148	0.1565
16	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920	0.2211
17	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784	0.2970
18	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695	0.3814
19	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606	0.4703
20	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472	0.5591
21	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255	0.6437
22	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931	0.7206
23	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490	0.7875
24	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933	0.8432
25	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269	0.8878

NORMAL DISTRIBUTION

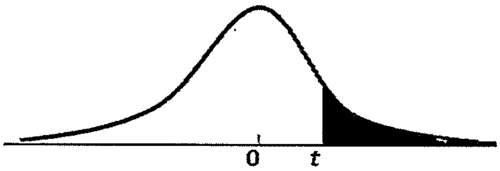
This table gives the area under the normal curve between the mean and a point Z standard deviations above the mean. The corresponding area for deviations below the mean can be found by symmetry.



$z = \frac{(x-\mu)}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0159	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4430	.4441
1.6	.4452	.4463	.4474	.4485	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4762	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4865	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4980	.4980	.4981
2.9	.4981	.4982	.4983	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	**.49865**	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	**.49903**	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	**.49931**	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	**.49952**	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	**.49966**	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	**.49977**									

T DISTRIBUTION TABLE

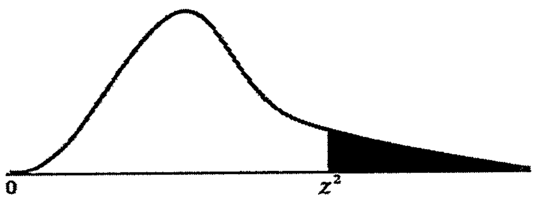
The critical values of t distribution are calculated according to the probabilities of two alpha values and the degrees of freedom. The Alpha(α) values 0.05 one tailed and 0.1 two tailed are the two columns to be compared with the degrees of freedom in the row of the table.



α (1 tail) α (2 tail) df	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.01	0.0025 0.005	0.001 0.002	0.0005 0.001
1	6.3138	12.7065	31.8193	63.6551	127.3447	318.4930	636.0450
2	2.9200	4.3026	6.9646	9.9247	14.0887	22.3276	31.5989
3	2.3534	3.1824	4.5407	5.8408	7.4534	10.2145	12.9242
4	2.1319	2.7764	3.7470	4.6041	5.5976	7.1732	8.6103
5	2.0150	2.5706	3.3650	4.0322	4.7734	5.8934	6.8688
6	1.9432	2.4469	3.1426	3.7074	4.3168	5.2076	5.9589
7	1.8946	2.3646	2.9980	3.4995	4.0294	4.7852	5.4079
8	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008	5.0414
9	1.8331	2.2621	2.8214	3.2498	3.6896	4.2969	4.7809
10	1.8124	2.2282	2.7638	3.1693	3.5814	4.1437	4.5869
11	1.7959	2.2010	2.7181	3.1058	3.4966	4.0247	4.4369
12	1.7823	2.1788	2.6810	3.0545	3.4284	3.9296	4.3178
13	1.7709	2.1604	2.6503	3.0123	3.3725	3.8520	4.2208
14	1.7613	2.1448	2.6245	2.9768	3.3257	3.7874	4.1404
15	1.7530	2.1314	2.6025	2.9467	3.2860	3.7328	4.0728
16	1.7459	2.1199	2.5835	2.9208	3.2520	3.6861	4.0150
17	1.7396	2.1098	2.5669	2.8983	3.2224	3.6458	3.9651
18	1.7341	2.1009	2.5524	2.8784	3.1966	3.6105	3.9216
19	1.7291	2.0930	2.5395	2.8609	3.1737	3.5794	3.8834
20	1.7247	2.0860	2.5280	2.8454	3.1534	3.5518	3.8495
21	1.7207	2.0796	2.5176	2.8314	3.1352	3.5272	3.8193
22	1.7172	2.0739	2.5083	2.8188	3.1188	3.5050	3.7921
23	1.7139	2.0686	2.4998	2.8073	3.1040	3.4850	3.7676
24	1.7109	2.0639	2.4922	2.7970	3.0905	3.4668	3.7454
25	1.7081	2.0596	2.4851	2.7874	3.0782	3.4502	3.7251
26	1.7056	2.0555	2.4786	2.7787	3.0669	3.4350	3.7067
27	1.7033	2.0518	2.4727	2.7707	3.0565	3.4211	3.6896
28	1.7011	2.0484	2.4671	2.7633	3.0469	3.4082	3.6739
29	1.6991	2.0452	2.4620	2.7564	3.0380	3.3962	3.6594
30	1.6973	2.0423	2.4572	2.7500	3.0298	3.3852	3.6459
31	1.6955	2.0395	2.4528	2.7440	3.0221	3.3749	3.6334
32	1.6939	2.0369	2.4487	2.7385	3.0150	3.3653	3.6218
33	1.6924	2.0345	2.4448	2.7333	3.0082	3.3563	3.6109
34	1.6909	2.0322	2.4411	2.7284	3.0019	3.3479	3.6008
35	1.6896	2.0301	2.4377	2.7238	2.9961	3.3400	3.5912
36	1.6883	2.0281	2.4345	2.7195	2.9905	3.3326	3.5822
37	1.6871	2.0262	2.4315	2.7154	2.9853	3.3256	3.5737
38	1.6859	2.0244	2.4286	2.7115	2.9803	3.3190	3.5657
39	1.6849	2.0227	2.4258	2.7079	2.9756	3.3128	3.5581
40	1.6839	2.0211	2.4233	2.7045	2.9712	3.3069	3.5510

CHI-SQUARE PROBABILITIES

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (i.e.: 0.05 on the left is 0.95 on the right)



The shaded area is equal to α *for* $\chi^2 = \chi^2_\alpha$

df	0.995	0.99	0.975	0.95	0.90	0.10	0.050	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

BINOMIAL COEFFICIENTS

x	0	1	2	3	4	5	6	7	8	9	10
n=1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

Example: $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Instructions for Candidates / अभ्यर्थियों के लिए निर्देश:

1. Write your Roll No. on the top immediately on receipt of this question paper / यह प्रश्न पत्र मिलते ही सबसे ऊपर अपना नामांक लिखें।
2. Answers may be written in English or Hindi but the same medium should be used throughout the paper / उत्तर अंग्रेजी या हिंदी में लिखे जा सकते हैं, परन्तु संपूर्ण प्रश्न पत्र में एक ही माध्यम का उपयोग करें।
3. The question paper consists of six questions / प्रश्न पत्र में छह प्रश्न हैं।
4. **Q1 is compulsory.** It carries 10 marks / प्रश्न 1 अनिवार्य है। इसके 10 अंक हैं।
5. **Do any four out of five questions from Q2 to Q6.** Each question carries 20 marks / प्रश्न 2 से प्रश्न 6 तक पाँच प्रश्नों में से कोई चार प्रश्न करें। प्रत्येक प्रश्न के 20 अंक हैं।
6. Use of simple non programmable calculator is allowed / सामान्य नॉन प्रोग्रामेबल कैलकुलेटर का उपयोग किया जा सकता है।
7. Statistical tables are attached for your reference / आपके संदर्भ हेतु सांख्यिकीय सारणी संलग्न है।

Q-1 Consider the multiple regression model:

$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

where the CLRM assumptions MLR.1 through MLR.4 hold (linearity, random sampling, no perfect collinearity, and zero conditional mean). Suppose the data are given as:

Y = (5,7,9,6,8), X1 = (1,0,2,1,3), X2 = (2,1,0,3,2), X3 = (4,5,3,2,1)

a) Write the regression model in matrix notation, clearly defining all matrices.

b) Using matrix algebra, calculate the OLS estimator $\beta = (X'X)^{-1}X'Y$.

c) Suppose we are interested in the linear combination $\theta = \beta_1 + \beta_2$:

(i) Show that $\theta = \beta_1 + \beta_2$ is an unbiased estimator of θ.

(ii) Derive Var(θ) in terms of Var(β_1), Var(β_2), and Cov(β_1, β_2).

d) Compute the Total Sum of Squares (TSS), Explained Sum of Squares (ESS), and Residual Sum of Squares (RSS) using matrix notation.

e) Find the coefficient of determination R^2 and interpret it. 5*2=10

प्रश्न -1 नीचे दिये गये बहु समाश्रयण प्रतिरूप पर विचार कीजिये:

$Yi = \beta 0 + \beta 1X1i + \beta 2X2i + \beta 3X3i + ui$

जहां CLRM धारणाएं MLR.1 से MLR.4 तक लागू होती हैं (रैखिक, यादृच्छिक प्रतिदर्शण, पूर्ण रैखिकता का न होना, शून्य प्रतिबंधात्मक माध्य)। मान लीजिए कि आंकड़े निम्नवत दिये गये हैं:

$Y = (5,7,9,6,8), X_1 = (1,0,2,1,3), X_2 = (2,1,0,3,2), X_3 = (4,5,3,2,1)$

a) सभी मैट्रिक्स को स्पष्टतः परिभाषित करते हुए, समाश्रयण प्रतिरूप को मैट्रिक्स संकेतन में लिखिये।

b) मैट्रिक्स अलजेब्रा का उपयोग करते हुए, ओएलएस प्राक्कलक $\beta = (X'X)^{-1}X'Y$ की गणना कीजिये।

c) मान लीजिए कि रैखिक संयोजन $\theta = \beta_1 + \beta_2$ में हमारी रूचि है:

(i) दर्शाइए कि $\theta = \beta_1 + \beta_2$, θ का अनभिनत प्राक्कलक है।

(ii) $\text{Var}(\beta_1)$, $\text{Var}(\beta_2)$, और $\text{Cov}(\beta_1, \beta_2)$ के संदर्भ में $\text{Var}(\theta)$ ज्ञात कीजिये।

(iii) मैट्रिक्स संकेतन का उपयोग करते हुए कुल वर्ग योगफल (TSS), व्याख्यायित वर्ग योगफल, (ESS), और अवशिष्ट वर्ग योगफल (RSS) की गणना कीजिये।

(iv) विनिर्धारण गुणांक R^2 ज्ञात कीजिये और उसका तात्पर्य समझाइये। 5*2=10

Q-2

a) Suppose a linear probability model (LPM) yields a predicted value of Y that is equal to −0.4. Explain why this is problematic.

b) Why are the coefficients of probit and logit models estimated using the method of maximum likelihood (MLE), rather than using the instrumental variables (IV) method?

c) Your friend is analyzing survey data on individuals' exercise habits to study what predicts regular physical activity in your city. She asks if she should use a logit, probit, or linear probability model. What are the advantages and disadvantages of each model in this context? Which would you recommend, and why?

d) Now suppose the dependent variable Yi represents the number of hours per week an individual exercises, but many individuals report zero hours. Explain why a Tobit model may be more appropriate in this case. Discuss how the Tobit model differs from the logit and probit models in both purpose and interpretation.

e) You are analyzing household spending on luxury goods, where some households report zero expenditure because they choose not to buy such goods. Describe how you could use a two-part model that combines a logit/probit model with an OLS or Tobit model to analyze this data. What does each stage of the model capture? 5*4 =20

प्रश्न-2

a) मान लीजिए रैखिक प्रायिकता प्रतिरूप (LPM) Y का अनुमानित मान प्रदान करता है जो −0.4 के समान है। समझाइएं कि यह समस्यात्मक क्यों है।

b) प्रोबिट और लॉगिट प्रतिरूप के गुणांक का अनुमान साधन चर (IV) पद्धति के बजाय अधिकतम संभाव्यता

(MLE) पद्धति का उपयोग करके क्यों लगाया जाता है?

c) आपकी मित्र आपके शहर में नियमित शारीरिक गतिविधियों का अनुमान लगाने के लिए लोगों की व्यायाम संबंधी आदतों के संबंध में सर्वेक्षण आंकड़ों का विश्लेषण कर रही है। वह पूछती है कि क्या उसे लॉगिट, प्रोबिट, या रैखिक प्रायिकता प्रतिरूप का उपयोग करना चाहिए। इस मामले में प्रत्येक प्रतिरूप के क्या फायदे और नुकसान हैं? आप उसके किस प्रतिरूप की सिफारिश करेंगे और क्यों?

d) अब मान लीजिए आश्रित चर Yi किसी व्यक्ति द्वारा प्रत्येक सप्ताह में किये गये व्यायाम के घंटों की संख्या को दर्शाता है, परन्तु कई लोग शून्य घंटे व्यायाम करना सूचित करते हैं। बताइएं कि इस मामले में टोबिट प्रतिरूप अधिक सटीक क्यों हो सकता है। चर्चा कीजिये कि टोबिट प्रतिरूप, उद्देश्य और व्याख्या दोनों में लॉगिट और प्रोबिट प्रतिरूप से किस प्रकार अलग है।

e) आप लग्ज़री सामान पर घरेलू खर्च का विश्लेषण कर रहे हैं, जहां कुछ घर शून्य खर्च होना सूचित करते हैं क्योंकि वे ऐसे सामान नहीं खरीदना पसंद करते हैं। बताइएं कि आप इस आंकड़े का विश्लेषण करने के लिए दो-भाग प्रतिरूप का उपययोग किस प्रकार कर सकते हैं जो लॉगिट/प्रोबिट प्रतिरूप को ओएलएस या टोबिट प्रतिरूप के साथ जोड़ता है। प्रतिरूप का प्रत्येक चरण क्या प्राप्त करता है? 5*4 =20

Q-3 Suppose the market for milk can be described by the following system:

$$Qs = \alpha 1 + \beta 1P + \gamma 1Zs + u1 \quad Qd = \alpha 2 + \beta 2P + \gamma 2Zd + u2$$

where Qs and Qd are quantity supplied and demanded, P is price, and Zs and Zd are exogenous supply and demand shifters respectively. In equilibrium, Qs = Qd = Q.

a) Define what is meant by a structural equation and a reduced form equation in this context.

b) Explain why OLS estimation of the supply curve generally produces biased estimates of β1.

c) Under what condition is the demand equation identified? What role does the variable Z's play in this identification?

d) Describe how the two-stage least squares (2SLS) method would be used to estimate the demand equation in this system. 4*5=20

Q-3 मान लीजिए कि दूध के बाज़ार को इस प्रणाली से व्याख्यायित किया जा सकता है:-

$$Q_s = \alpha_1 + \beta_1 P + \gamma_1 Z_s + u_1$$
$$Q_d = \alpha_2 + \beta_2 P + \gamma_2 Z_d + u_2$$

जहाँ Q_s और Q_d आपूर्तित और की गयी मांग की मात्रा है, P कीमत है, और Z_s और Z_d क्रमशः बहिर्जात आपूर्ति और मांग परिवर्तन कारक हैं। समतुल्यता में, $Q_s = Q_d = Q$ है।

a) इस संदर्भ में संरचनात्मक समीकरण और समानीत रूप समीकरण से क्या तात्पर्य है, परिभाषित कीजिये।

b) आपूर्ति वक्र का ओएलएस प्राक्कलन आम तौर पर β1 का अभिनत प्राक्कलन क्यों प्रदान करता है, समझाइये।

c) किस शर्त के तहत मांग समीकरण की पहचान की जाती है? इस पहचान में चर Z's की क्या भूमिका है?

d) इस प्रणाली में मांग समीकरण के प्राक्कलन के लिये द्वि-चरण न्यूनतम वर्ग (2SLS) पद्धति का उपयोग किस प्रकार किया जा सकता है। 4*5=20

Q-4 Consider a school-level analysis examining the impact of health insurance coverage on healthcare utilization among high school students, modelled as follows:

$$H_s = \beta 0 + \beta 1 I_c + \beta 2\ FP + \beta 3 A_V + \beta 4 urban + u1 \qquad \text{Equation (1)}$$

where:

H_S= percentage of students who visited a hospital in the past year,

I_C= percentage of students with health insurance coverage,

F_P= percentage of female students in the school population.

A_V = average family income at the school,

urban = dummy variable indicating whether a school is in an urban area.

The model is estimated at the school level.

a) Interpreting the preceding Equation 1 in a causal, ceteris paribus fashion, what is the expected sign of β1? Explain the intuition behind this expected sign in the context of healthcare access and insurance coverage.

b) Why might H_S and I_C be jointly determined? Describe the potential feedback mechanisms that could create reverse causality between these two variables. In particular, discuss how healthcare utilization could influence the decision or likelihood of insurance enrolment, rather than operating only in the forward causal direction.

c) Suppose the true relationship determining insurance coverage is:

$$I_C = \gamma 0 + \gamma 1 H_S + \text{other factors}, \qquad \text{Equation (2)}$$

and assume $\gamma 1 > 0$, meaning that higher hospital utilization rates increase the likelihood of insurance enrolment. What is the likely bias in estimating $\beta 1$ by OLS?

d) Suppose there exists a binary variable, G_P, which equals 1 if a school participates in a government or private health insurance assistance program, and 0 otherwise. Explain how G_P can be used as an instrumental variable (IV) to estimate $\beta 1$ (and the other parameters) in the Equation

(1). What conditions must G_P satisfy to serve as a valid instrument in this context? Specifically, discuss its relevance and exogeneity with respect to both the hospital visits and insurance rate equations. 4*5=20

प्रश्न 4 हाई विद्यालय के विद्यार्थियों के बीच स्वास्थ्य देखभाल के उपयोग के संबंध में स्वास्थ्य बीमा व्याप्ति के प्रभाव की जांच करने वाले विद्यालयी-स्तर के विश्लेषण पर विचार कीजिये, जिसे निम्नवत प्रतिरूपित किया गया है:

$$H_s = \beta_0 + \beta_1 I_c + \beta_2 F_P + \beta_3 A_v + \beta_4 urban + u_1 \qquad \text{Equation (1)}$$

जहां:

H_s= पिछले वर्ष अस्पताल जाने वाले विद्यार्थियों का प्रतिशत है,

I_c= स्वास्थ्य बीमा व्याप्ति वाले विद्यार्थियों का प्रतिशत,

F_p= विद्यालय की आबादी में बालिका विद्यार्थियों का प्रतिशत,

A_v = विद्यालय में परिवार की औसत आय।

urban = यह मूक चर है जो बताता है कि विद्यालय शहरी क्षेत्र में है अथवा नहीं।

इस प्रतिरूप का प्राक्कलन विद्यालयी स्तर पर किया गया है।

a) पिछले समीकरण 1 को हेतुक के रूप में व्याख्यायित करते हुए, यदि अन्य बातें पूर्ववत रहने पर, β1 का अपेक्षित प्रतीक क्या है? स्वास्थ्य देखभाल पहूंच और बीमा व्याप्ति के संदर्भ में इस अपेक्षित प्रतीक के पीछे की समझ की व्याख्या कीजिए।

b) H_s और I_c को एक साथ क्यों निर्धारित किया जा सकता है? उन संभावित फीडबैक तंत्र का वर्णन कीजिये जो इन दो चरों के बीच व्युत्क्रम हेतुक का निर्माण कर सकते हैं। विशेषतः, इस बात पर चर्चा कीजिये कि स्वास्थ्य देखभाल का उपयोग केवल अग्र हेतुक दिशा में कार्य करने के बजाय, बीमा नामांकन के निर्णय या की संभावना को किस प्रकार प्रभावित कर सकता है।

c) मान लीजिए कि बीमा व्याप्ति को निर्धारित करने वाला असली संबंध निम्नवत है:

$I_c = \gamma_0 + \gamma_1 H_s + other\ factors,$ Equation (2)

और मान लीजिये कि $\gamma_1 > 0$, जिसका तात्पर्य है कि अस्पताल के उपयोग की अधिक दर बीमा नामांकन की संभावना को बढ़ा देती है। ओएलएस द्वारा β1 का प्राक्कलन करते समय संभावित अभिनति क्या है?

d) मान लीजिए कि कोई द्विचर G_p, है, जो किसी विद्यालय के सरकारी या निजी स्वास्थ्य बीमा सहायता कार्यक्रम में भाग लेने पर 1 है अन्यथा 0 है। समझाइएं कि समीकरण (1) में β1 (और दूसरे प्राचलों) का प्राक्कलन करने के लिए G_p का उपयोग साधन चर (IV) के तौर पर किस प्रकार किया जा सकता है। इस संदर्भ में किसी वैध साधन के तौर पर कार्य करने हेतु G_p को कौन से प्रतिबंध को पूर्ण करना होगा? विशेषतः, अस्पताल जाने और बीमा दर समीकरणों, दोनों के संबंध में इसकी प्रासंगिकता और बहिर्जातता पर चर्चा कीजिये। 4*5=20

Q-5

(a) Explain the consistency and efficiency properties of the Fixed Effects (FE) estimator in panel data models. Under what conditions is the FE estimator unbiased and consistent?

(b) What is the “within transformation” used in the Fixed Effects model? Explain how it helps in eliminating unobserved individual-specific effects.

(c) What is a “balanced panel” vs. an “unbalanced panel”? How does it affect estimation?

(d) What are the limitations of using panel data models with short time periods (small T, large N)? 4*5=20

प्रश्न 5

(a) पैनल डेटा प्रतिरूपों में निश्चित प्रभाव(FE) प्राक्कलक की निरंतरता और दक्षता गुणों को समझाइएं। किन प्रतिबंधों के तहत निश्चित प्रभाव प्राक्कलक अनभिनत और निरंतर होता है?

(b) निश्चित प्रभाव प्रतिरूप में उपयोग किये जाने वाले “भीतर रूपांतरण” क्या है? यह अनअवलोकित वैयक्तिक-विशिष्ट प्रभाव को समाप्त करने में किस प्रकार सहायता करता है।

(c) “संतुलित पैनल” बनाम “असंतुलित पैनल” क्या है? यह प्राक्कलन को किस प्रकार प्रभावित करता है?

(d) पैनल डेटा प्रतिरूपों को लघु समायावधि (T लघु, N दीर्घ) में उपयोग करने में क्या सीमाएं हैं? 4*5=20

Q-6 Consider the following infinite distributed lag model:

$$Yt = \alpha + \beta 0Xt + \beta 1Xt{-}1 + \beta 2Xt{-}2 + \beta 3Xt{-}3 + \cdots + \mu t$$

a) Show how the Koyck transformation can be used to transform the distributed lag model given above into an estimable form of an AR model.

b) What restrictions does Koyck impose on the λ, where λ is the rate of decline of the distributed lag : $\beta k = \beta 0\lambda^k$, k=0, 1, 2, 3...

c) Write the three features of the Koyck scheme.

d) Prove that for the Koyck model, the mean lag is $\frac{\lambda}{1-\lambda}$

प्रश्न 6 नीचे दिए गए अनंत वितरित लैग प्रतिरूप पर विचार कीजिये:

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \cdots + \mu_t$$

a) दर्शाइए कि कोयक रूपांतरण का उपयोग ऊपर दिए गए वितरित लैग प्रतिरूप को एआर प्रतिरूप के प्राक्कलनीय रूप में रूपांतरित करने के किस प्रकार किया जा सकता है।

b) कोयक λ पर क्या अवरोध लगाता है, जहाँ λ वितरित लैग $\beta_k = \beta_0\lambda^k$, k=0, 1, 2, 3... के घटने की दर है।

c) कोयक स्कीम के तीन विशेषताएं लिखिए।

d) सिद्ध कीजिये कि कोयक प्रतिरूप हेतु, माध्य लैग $\frac{\lambda}{1-\lambda}$ है।